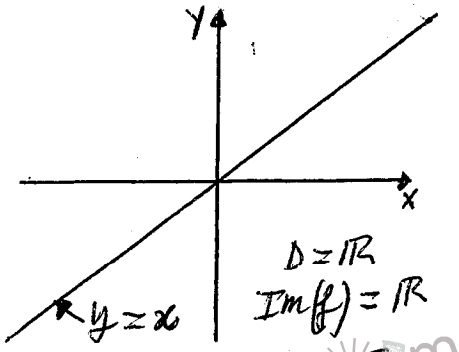
PREPARAZIONE ESAME SCRITTO ANALISI

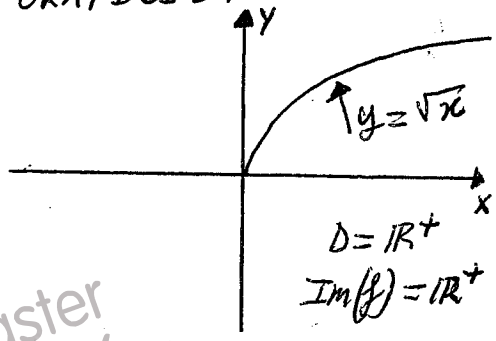
SCHEDE RIASSUNTIVE

GRAFICI E FUNZIONI



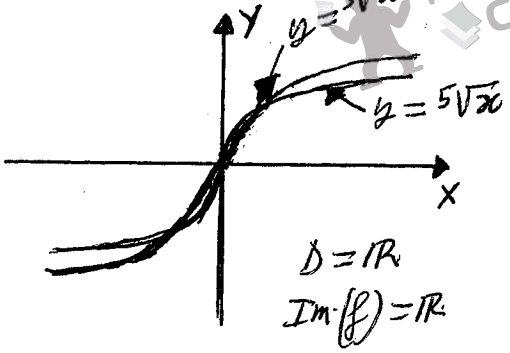
$D = \mathbb{R}$

$Im(f) = \mathbb{R}$



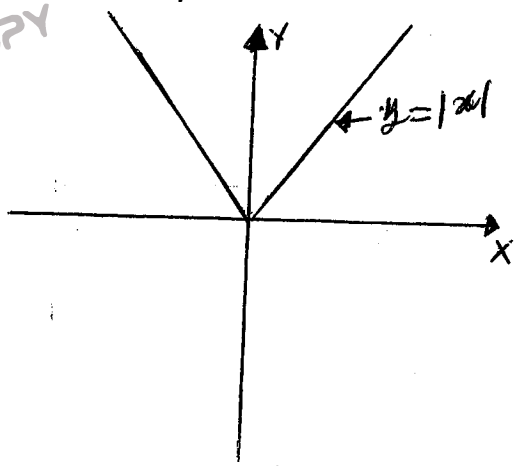
$D = \mathbb{R}^+$

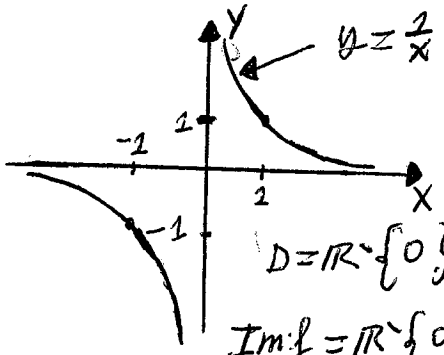
$Im(f) = \mathbb{R}^+$



$D = \mathbb{R}$

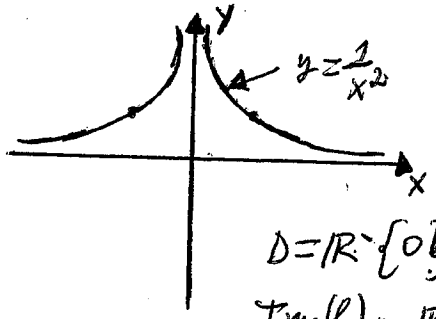
$Im(f) = \mathbb{R}$





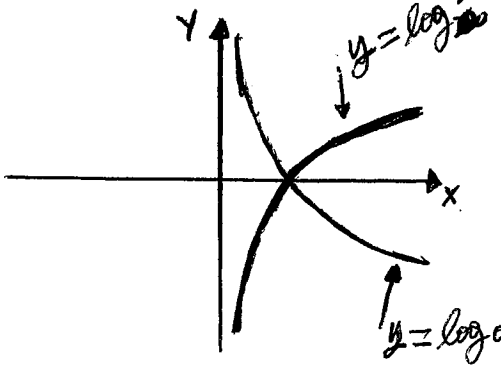
$D = \mathbb{R} \setminus \{0\}$

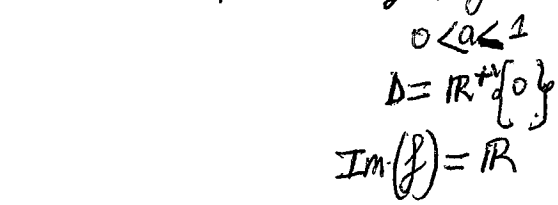
$Im(f) = \mathbb{R} \setminus \{0\}$



$D = \mathbb{R} \setminus \{0\}$

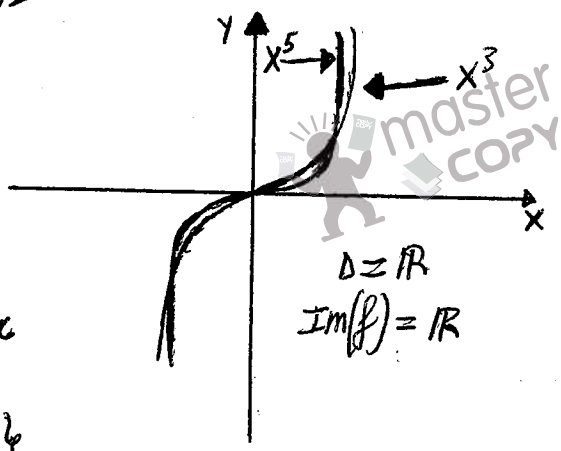
$Im(f) = \mathbb{R}^+ \cup \{0\}$





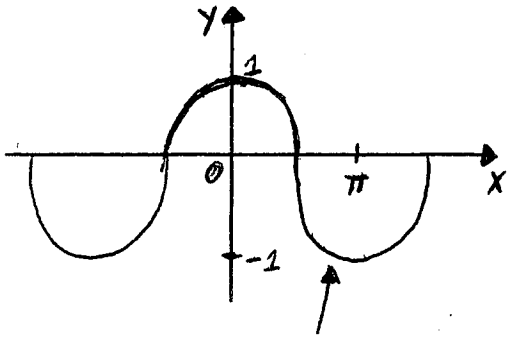
$D = \mathbb{R}^+ \setminus \{0\}$

$Im(f) = \mathbb{R}$



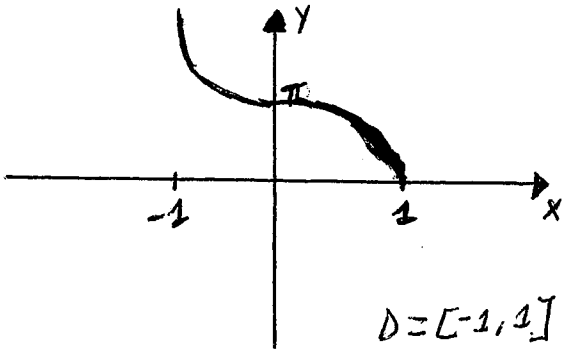
$D = \mathbb{R}$

$Im(f) = \mathbb{R}$



$D = \mathbb{R}$

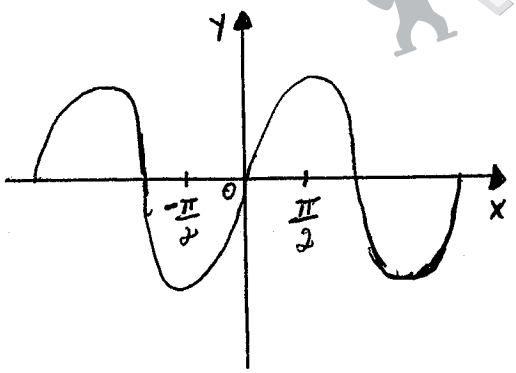
$Im(f) = [-1, 1]$

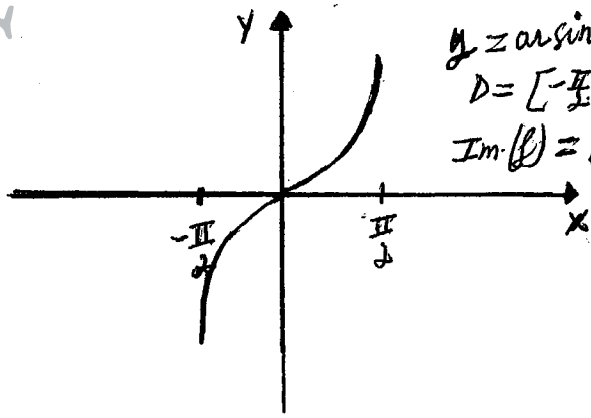


$D = [-1, 1]$

$Im(f) = [0, \pi]$

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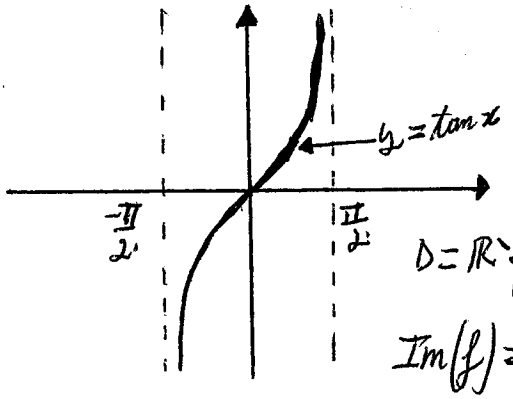




$y = \arcsin x$

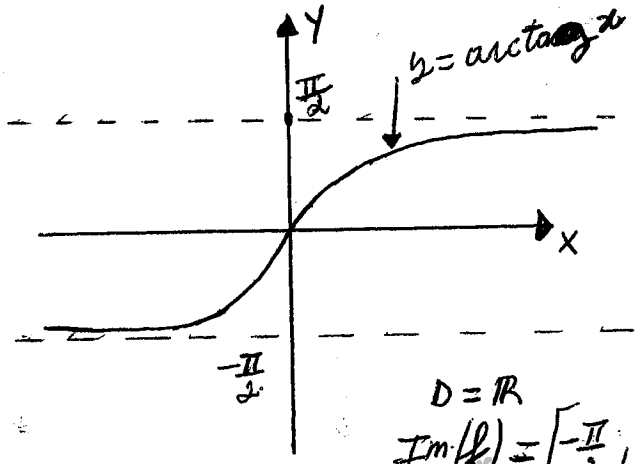
$D = [-\frac{\pi}{2}, \frac{\pi}{2}]$

$Im(f) = [-1, 1]$



$D = \mathbb{R} \setminus \{k\frac{\pi}{2}\}$

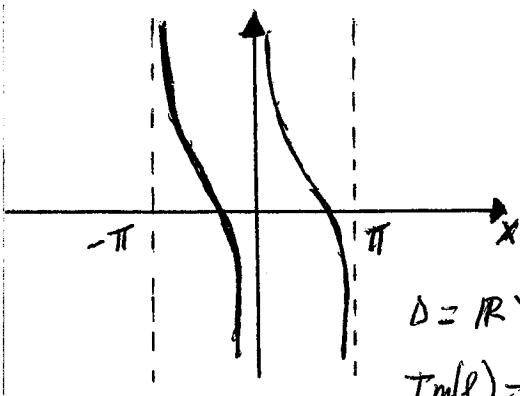
$Im(f) = \mathbb{R}$



$D = \mathbb{R}$

$Im(f) = [-\frac{\pi}{2}, \frac{\pi}{2}]$

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$D = \mathbb{R} \setminus \{k\pi\}$

$Im(f) = \mathbb{R}$

Limiti

Forme indeterminate	Non forme indeterminate
• $+\infty + (-\infty)$ "$+\infty \cdot 0$" "$-\infty \cdot 0$" $\frac{\pm\infty}{\pm\infty}$; $\frac{0}{0}$; $\frac{c}{0}$	$+\infty + \infty = +\infty$ $+\infty - \infty = -\infty$ $\frac{c}{0^+} = +\infty$ $\frac{c}{0^-} = -\infty$ $\frac{l}{\pm\infty} = 0$

Confronto infinito / infinitesimo

$f(x) \ll g(x) \longrightarrow \lim_{x \to x_0} \frac{f(x)}{g(x)} = 0$

$f(x) = o(g(x))$

per $x \longrightarrow +\infty$ (Contrario per $x \to 0$)

- $x^a \ll x^b$ se $a < b$
- $a^x \ll b^x$ se $a < b$ $a > 0$
- $\log \ll x^a$ se $a > 0$ se $a < 0$ $x^a \ll \log x$
- $x^a \ll b^x$ se $b > 1$

- $o(g) + o(g) = o(g)$
- $f \sim g$ se $\lim_{x \to x_0} \frac{f(x)}{g(x)} = 1$
- $(\log x)^a \ll x^b$ se $b > 0$
- $f \sim g \iff f = g + o(g)$

Se $\exists L = \lim_{x \to x_0} \frac{|f(x)|}{|g(x)|}$ allora $f(x) = O(g(x))$

- $f(x) \sim ax^b$ per $x \longrightarrow 0$ opp $x \longrightarrow +\infty$

ax^b (parte principale di $f(x)$ (per $x \to 0$ opp $x \to +\infty$)

Se $f \sim g$ allora $\lim_{x \to x_0} f(x) = \lim_{x \to x_0} g(x)$ (Se non $\exists$ uno anche l'altro non $\exists$)

$f(x) \longrightarrow -f(x)$ riflessione rispetto all'asse delle x

$f(x) \longrightarrow f(-x)$ riflessione rispetto all'asse delle y

$f(x) \longrightarrow a\,f(x)$ compressione orizzontale di un fattore a

$f(x) \longrightarrow f(ax)$ compressione verticale di un fattore a

$f(x) \longrightarrow f(|x|)$ parte a destra dell'asse delle x + la riflessione rispetto all'asse delle y

$f(x) \longrightarrow |f(x)|$ ribaltamento della parte del grafico sotto l'asse delle x

$f(x) \longrightarrow f(x+a)$ traslazione orizzontale verso
- sinistra di $+a$ se $a>0$
- destra di $-a$ se $a<0$

$f(x) \longrightarrow f(x)+a$ traslazione verticale di a
- verso l'alto se $a>0$
- verso il basso se $a<0$

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derivate di funzioni

$f(x)$	$f'(x)$
(costante) $a \in \mathbb{R}$	0
$ax+b$ $a,b \in \mathbb{R}$	a
x^a $a \neq 0$	$a\,x^{a-1}$
e^x	e^x
a^x	$\log a\, a^x$
$\log(x)$	$\frac{1}{x}$
$\sin x$	$\cos x$
$\cos x$	$-\sin x$
$\tan x$	$\frac{1}{\cos^2 x} = 1+\tan^2 x$
$\arcsin x$	$\frac{1}{\sqrt{1-x^2}}$
$\arccos x$	$-\frac{1}{\sqrt{1-x^2}}$
$\arctan x$	$\frac{1}{1+x^2}$
$\cot x$	$-\frac{1}{\sin^2 x}$
$f+g$	$f'+g'$
$f-g$	$f'-g'$
$f \cdot g$	$f'g+fg'$
$\frac{f}{g}$	$\frac{f'g-g'f}{g^2}$
$f(g(x))$	$f'(g(x)) \cdot g'(x)$
$[g(x)]^{v(x)}$	$[g(x)]^{v(x)}\left[v'(x)\cdot\log g(x) + \frac{v(x)\cdot g'(x)}{g(x)}\right]$

$f(x) = \sinh(x) = \frac{e^x - e^{-x}}{2}$

$f(x) = \cosh(x) = \frac{e^x + e^{-x}}{2}$

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$f_1 \sim g_1$ e $f_2 \sim g_2$ $\quad f_1 \cdot f_2 \sim g_1 \cdot g_2$

$$\frac{f_1}{f_2} \sim \frac{g_1}{g_2}$$

$$\longrightarrow \lim_{x \to x_0} f_1(x) f_2(x) = \lim_{x \to x_0} g_1(x) \cdot g_2(x)$$

$$\lim_{x \to x_0} \frac{f_1(x)}{f_2(x)} = \lim_{x \to x_0} \frac{g_1(x)}{g_2(x)}$$

attenzione $f_1 \sim g_1$ e $f_2 \sim g_2 \not\longrightarrow f_1 + f_2 \sim g_1 + g_2$

Se $f_1 \ll g$ e $f_2 \ll g$ con $c_1, c_2 \in \mathbb{R}$ allora

$c_1 f_1 + c_2 f_2 \ll g$

$c_1 o(g) + c_2 o(g) = o(g)$ $\quad f_1 \sim f_2 \not\longrightarrow f_1 = f_2$

Se $f(x) = o(g(x))$ per $x \to x_0$ allora $f(x) = O(g(x))$

Se $f(x) = o(g(x))$ ordine $f <$ ordine di g

$f(x) = O(g(x))$ ordine $f \leq$ ordine g

- se $f(x) = O(x^a)$ e $a \leq b \longrightarrow f(x) = O(x^b)$
- se $f(x) = O(g(x))$ e $g(x) = O(h(x))$ allora $f(x) = O(h(x))$
- se $f(x) = O(x^a)$ e $a < b$ allora $f(x) = o(x^b)$
- se $f(x) = O(g(x))$ e $g(x) = o(h(x))$ allora $f(x) = o(h(x))$
- se $f(x) = O(x^a)$ e $b \leq a$ allora $f(x) = O(x^b)$
- se $f(x) = o(x^a)$ e $b < a$ allora $f(x) = o(x^b)$

una funzione f è continua in x_0 se $f(x_0) = \lim_{x \to x_0^{\pm}} f(x)$

f è derivabile in x_0 se

$$\lim_{x \to x_0^+} f'(x) = \lim_{x \to x_0^-} f'(x) = k \neq \pm\infty$$