Lezione 01 — Analisi II — 3/3/21

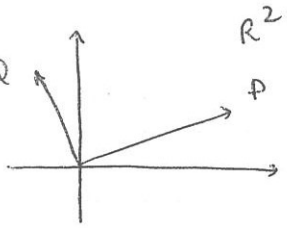
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Argomenti:

- calcolo differenziale in $\mathbb{R}^n$
- calcolo integrale in $\mathbb{R}^n$
- superfici e curve in $\mathbb{R}^n$
- Funzioni implicite
- Teoremi di Stokes

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Spazio Euclideo $\mathbb{R}^n$



$P, Q \in \mathbb{R}^n$

$$d(P,Q) = \sqrt{(p_1-q_1)^2 + \cdots + (p_n-q_n)^2}$$

$P = (p_1, \ldots, p_n)$

$Q = (q_1, \ldots, q_n)$

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Proprietà di d

1) $d(P,Q) \geq 0 \quad \forall P, Q$
2) $d(P,Q) = 0 \iff P = Q$
3) $d(P,Q) = d(Q,P)$
4) $d(P,Q) \leq d(P,R) + d(R,Q) \quad \forall P,Q,R$ disug. triang.

$\mathbb{R}^n$ metrico completo $\{P_N\}$:

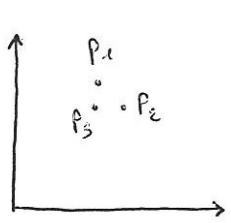
$$\boxed{\forall \varepsilon > 0 \quad \exists N > 0 \quad \text{t.c. } \forall n, m > N \quad d(P_N, P_M) < \varepsilon}$$

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$\Rightarrow \exists \bar{P} \quad d(P_n, \bar{P}) \xrightarrow{N \to +\infty} 0$ ← converge

CAUCHY

$\forall \varepsilon' \quad \exists N' > 0 \quad \forall n > N' \quad d(P_N, \bar{P}) < \varepsilon'$



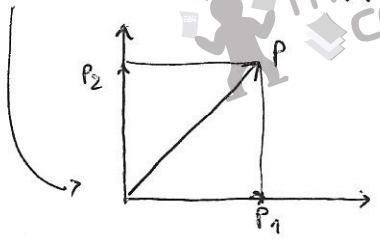
$\forall \varepsilon > 0 \quad N \quad \forall m, m > N$

$d(P_N, P_M) < \varepsilon$

PRODOTTO SCALARE

$P, Q \qquad \langle P, Q \rangle = \sum_{i=1}^{N} P_i Q_i$

$\|P\| = \sqrt{\langle P, P \rangle} = \sqrt{\sum_{i=1}^{N} P_i^2} = \sqrt{P_1^2 + P_2^2 + \dots + P_N^2}$



caso bidimensionale

Disug. di Cauchy - Schwartz

$$|\langle P, Q \rangle| \leq \|P\| \cdot \|Q\|$$

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Dim

$\|P + \lambda Q\|^2 = \langle P + \lambda Q, P + \lambda Q \rangle \geq 0 \quad \lambda \in R$

$\langle A, \lambda B \rangle = \lambda \langle A, B \rangle$

$\langle A, B + C \rangle = \langle A, B \rangle + \langle A, C \rangle$

$\langle P, P \rangle + 2\lambda \langle P, Q \rangle + \lambda^2 \langle Q, Q \rangle \geq 0$

↳ vero per ogni λ

↳ significa che $\Delta \leq 0$

$\Delta = 4(\langle P, Q \rangle)^2 - 4(\langle P, P \rangle \langle Q, Q \rangle)$

$(\langle P, Q \rangle)^2 - \langle P, P \rangle \langle Q, Q \rangle \leq 0$

$(\langle P, Q \rangle)^2 \leq \langle P, P \rangle \langle Q, Q \rangle$

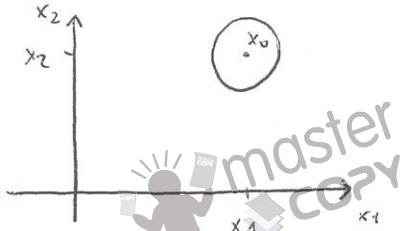
$|\langle P, Q \rangle| \leq \langle P, P \rangle^{1/2} \langle Q, Q \rangle^{1/2}$

$\langle P, P \rangle^{1/2} = \|P\| \qquad \langle Q, Q \rangle^{1/2} = \|Q\|$

Palle di raggio R centrate in

$x_0 \in \mathbb{R}^n \quad x_0 = (x_1, \dots, x_n)$

$B_R(x_0) = \{ y \in \mathbb{R}^n \text{ di } (x_0, y) \leq R \}$

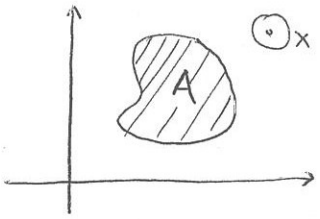


$(x_1 - y_1)^2 + (x_2 - y_2)^2 \leq R^2$

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Def

$x \in \mathbb{R}^n$ di accumulazione rispetto ad un insieme A



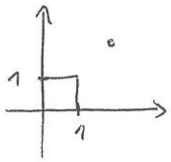
$\forall R > 0$

$B_R(x) \setminus \{x\} \cap A \neq \phi$

Def

$x \in A$ non di accumulazione si dice isolato

$A = [0,1] \times [0,1] \cup \{(2,2)\}$



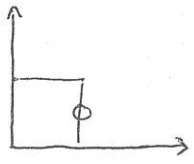
Def

A chiuso se

... $Acc(A) = \{ x \text{ punto di accumulazione di } A \}$

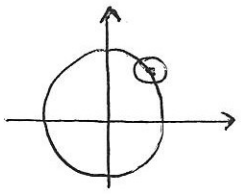
$Acc(A) \subseteq A$

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$[0,1] \times [0,1] \cup \{(2,2)\} = A$

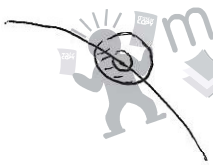
$A = \{(x, y) \in \mathbb{R}^2 \quad x^2 + y^2 < 1\}$



A non è chiuso

$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \notin A$

$Acc(A) \subseteq A$ è di accumulazione



$\bar{A}$ = chiusura di $A = A \cup Acc(A)$

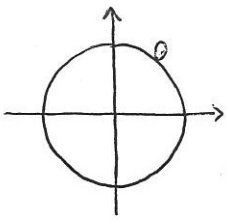
Def

A è aperto se $\forall x \in A \quad \exists R \quad B_R(x) \subset A$

PROP

A aperto $\iff$ A^c chiuso

$\boxed{A^c = \mathbb{R}^n \setminus A}$



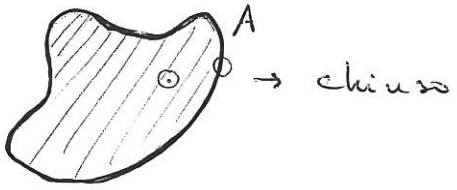
$\{x^2 + y^2 \leq 1\} = A$

$\{x^2 + y^2 > 1\} = A^c$

Def

frontiera di A, ∂A

$\partial A = \{x \in \mathbb{R}^n : x \in \bar{A} \text{ e } x \in \bar{A}^c\}$



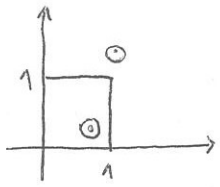
→ chiuso

$\forall R \quad B_R(x) \setminus \{x\} \cap A \neq 0$

$B_R(x) \setminus \{x\} \cap A^c \neq 0$

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$$A = \{(x,y) \in \mathbb{R}^2 \quad x,y \in \mathbb{Q} \quad 0 \le x \le 1, \ 0 \le y \le 1\}$$



$\overline{A} = [0,1] \times [0,1]$

$\overline{A}^c = A^c \cup Acc(A^c) = \mathbb{R}^2$

$\partial A = \overline{A} \cap A^c = [0,1] \times [0,1] \times \mathbb{R}^2$

$\left(\frac{\sqrt{2}}{2}, \frac{\sqrt{2}}{2}\right) \notin A$

$\left(\frac{1}{2}, \frac{1}{2}\right) \in A$

$[a,b] \times [c,d] = \{(x,y) \quad a \le x \le b, \ c \le y \le d\}$

$[0,1] \times [0,1]$

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