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ESAMI SVOLTI SdC II

2019

12/01

2018

17/11
15/09
21/07
30/06
09/06
14/04
17/02
03/02
13/01

2017

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2016

02/04
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2015

24/10

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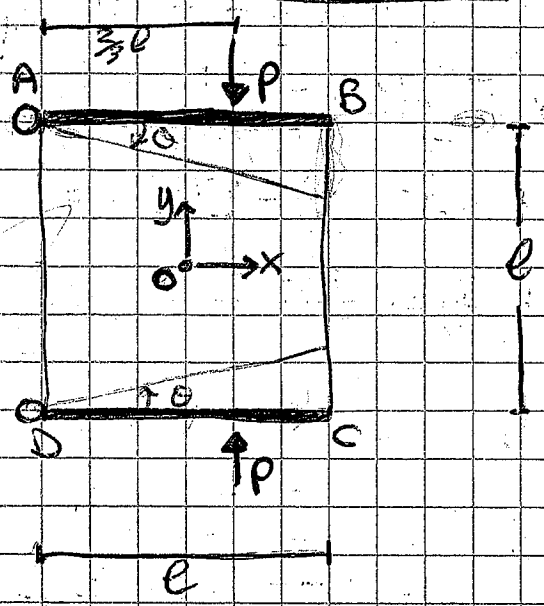
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ESAME 17/11/18

Problema 1

- problema piano
- materiale di Lamé
- no attrito
- campo di spostamento:

$$\begin{cases} u = b\left(x + \frac{x^2}{e}\right) + a\left(\frac{e}{2} - \frac{y^2}{e}\right) \\ v = ay\left(1 + \frac{2x}{e}\right) \end{cases}$$



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(a) Si deve:

$$\begin{cases} v\left(x, \frac{e}{2}\right) = -\theta\left(x + \frac{e}{2}\right) & \text{su } AB \\ v\left(x, -\frac{e}{2}\right) = \theta\left(x + \frac{e}{2}\right) & \text{su } DC \end{cases}$$

→ condizioni di vincolo imposte dalle due travi rigide

$$v\left(x, \frac{e}{2}\right) = a\frac{e}{2}\left(1 + \frac{2x}{e}\right) = -\theta\left(x + \frac{e}{2}\right) \rightarrow \boxed{a = -\theta}$$

$\boxed{b = \text{qualunque}}$ → vi corrispondono campi di spostamento cinematicamente ammissibili

(b) Componenti di deformazione:

$$\begin{cases} \varepsilon_x = u_{,x} = \frac{\partial}{\partial x}\left[b\left(x + \frac{x^2}{e}\right) + a\left(\frac{e}{2} - \frac{y^2}{e}\right)\right] = b\left(1 + \frac{2}{e}x\right) \\ \varepsilon_y = v_{,y} = \frac{\partial}{\partial y}\left[ay\left(1 + \frac{2x}{e}\right)\right] = a\left(1 + \frac{2}{e}x\right) = -\theta\left(1 + \frac{2}{e}x\right) \\ \gamma_{xy} = u_{,y} + v_{,x} = -a\frac{2y}{e} + a\frac{2y}{e} = 0 \end{cases}$$

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$$\varepsilon_x = \frac{1}{E}\left[\sigma_x - \nu\sigma_y\right] \rightarrow \sigma_x = E\varepsilon_x + \nu\sigma_y$$

$$\varepsilon_y = \frac{1}{E}\left[\sigma_y - \nu\sigma_x\right] \rightarrow \sigma_y = E\varepsilon_y + \nu\sigma_x \rightarrow \sigma_x = \frac{1}{\nu}\left[\sigma_y - E\varepsilon_y\right]$$

$$\gamma_{xy} = \frac{1}{G}\tau_{xy} \rightarrow \tau_{xy} = G\cdot\gamma_{xy} = 0$$

$$\sigma_x = E\varepsilon_x + \nu\sigma_y = \frac{1}{\nu}\left[\sigma_y - E\varepsilon_y\right]$$

$$\hookrightarrow E\left[b\left(1+\frac{2}{e}x\right)\right] + \nu\sigma_y = \frac{1}{\nu}\left[\sigma_y + E\theta\left(1+\frac{2}{e}x\right)\right]$$

$$\hookrightarrow \sigma_y\left(\nu - \frac{1}{\nu}\right) = \frac{E\theta}{\nu}\left(1+\frac{2}{e}x\right) - Eb\left(1+\frac{2}{e}x\right)$$

$$\hookrightarrow \sigma_y = \frac{E\left(1+\frac{2}{e}x\right)\left(\frac{\theta}{\nu}-b\right)}{\left(\nu-\frac{1}{\nu}\right)} =$$

$$= \left(\frac{\nu}{\nu^2-1}\right)\left[E\left(1+\frac{2}{e}x\right)\left(\frac{\theta}{\nu}-b\right)\right] =$$

$$\sigma_y = \frac{E}{1-\nu^2}\left(\nu b - \theta\right)\left(1+\frac{2}{e}x\right)$$

Componenti di tensione

$$\begin{cases} \sigma_x = \dfrac{E}{1-\nu^2}\left(b - \nu\theta\right)\left(1+\dfrac{2}{e}x\right) \\ \sigma_y = \dfrac{E}{1-\nu^2}\left(\nu b - \theta\right)\left(1+\dfrac{2}{e}x\right) \\ \tau_{xy} = 0 \end{cases}$$

c) Deve essere $\operatorname{div} T + b = 0$:

$$\begin{cases} \sigma_{x,x} + \tau_{xy,y} + \tau_{xz,z} + b_x = 0 \\ \tau_{yx,x} + \sigma_{y,y} + \tau_{yz,z} + b_y = 0 \\ \tau_{zx,x} + \tau_{zy,y} + \sigma_{z,z} + b_z = 0 \end{cases}$$

$$\sigma_{x,x} = 0 \rightarrow \frac{E}{1-\nu^2}\left(b-\nu\theta\right)\frac{2}{e} = 0 \rightarrow \boxed{b = \nu\theta}$$

$$\sigma_{y,y} = 0 \rightarrow 0 = 0$$

① $\begin{cases} \sigma_x = 0 \\ \sigma_y = \frac{E}{1-\vartheta^2}(\vartheta^2\theta - \theta)\left(1+\frac{2}{e}x\right) = -E\theta\left(1+\frac{2}{e}x\right) \\ \tau_{xy} = 0 \end{cases}$

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AB $\begin{cases} y = \frac{e}{2} \\ -\frac{e}{2} \le x \le \frac{e}{2} \end{cases}$ $n_{AB} = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$

$T \cdot n_{AB} = f^{AB} \rightarrow \begin{pmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} p_x^{AB} \\ p_y^{AB} \end{pmatrix} \rightarrow \begin{cases} p_x^{AB} = \tau_{xy}\left(x, \frac{e}{2}\right) = 0 \\ p_y^{AB} = \sigma_y\left(x, \frac{e}{2}\right) = -E\theta\left(1+\frac{2}{e}x\right) \end{cases}$

BC $\begin{cases} x = \frac{e}{2} \\ -\frac{e}{2} \le y \le \frac{e}{2} \end{cases}$ $n_{BC} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$

$T \cdot n_{BC} = f^{BC} \rightarrow \begin{pmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} p_x^{BC} \\ p_y^{BC} \end{pmatrix} \rightarrow \begin{cases} p_x^{BC} = \sigma_x\left(\frac{e}{2}, y\right) = 0 \\ p_y^{BC} = \tau_{xy}\left(\frac{e}{2}, y\right) = 0 \end{cases}$

CD $\begin{cases} y = -\frac{e}{2} \\ -\frac{e}{2} \le x \le \frac{e}{2} \end{cases}$ $n_{CD} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$

$T \cdot n_{CD} = f^{CD} \rightarrow \begin{pmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{pmatrix}\begin{pmatrix} 0 \\ -1 \end{pmatrix} = \begin{pmatrix} p_x^{CD} \\ p_y^{CD} \end{pmatrix} \rightarrow \begin{cases} p_x^{CD} = -\tau_{xy}\left(x, -\frac{e}{2}\right) = 0 \\ p_y^{CD} = -\sigma_y\left(x, -\frac{e}{2}\right) = +E\theta\left(1+\frac{2}{e}x\right) \end{cases}$

AD $\begin{cases} x = -\frac{e}{2} \\ -\frac{e}{2} \le y \le \frac{e}{2} \end{cases}$ $n_{AD} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$

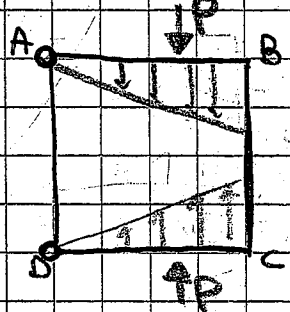
$T \cdot n_{AD} = f^{AD} \rightarrow \begin{pmatrix} \sigma_x & \tau_{xy} \\ \tau_{xy} & \sigma_y \end{pmatrix}\begin{pmatrix} -1 \\ 0 \end{pmatrix} = \begin{pmatrix} p_x^{AD} \\ p_y^{AD} \end{pmatrix} \rightarrow \begin{cases} p_x^{AD} = -\sigma_x\left(-\frac{e}{2}, y\right) = 0 \\ p_y^{AD} = -\tau_{xy}\left(-\frac{e}{2}, y\right) = 0 \end{cases}$

② **AB** Equilibrio al momento in A:

$P \cdot \frac{2}{3}\ell + \int_{-\frac{e}{2}}^{\frac{e}{2}} p_y^{AB}\,dx \cdot \frac{2}{3}\ell = 0$

$P - \int_{-\frac{e}{2}}^{\frac{e}{2}} E\theta\left(1+\frac{2}{e}x\right)dx = 0$

$p - E\theta e = 0 \rightarrow \theta = \frac{p}{Ee}$



su CD vive $\theta = -\frac{p}{Ee}$