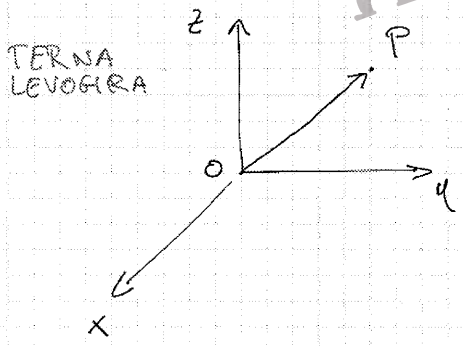
Grandezze vettoriali

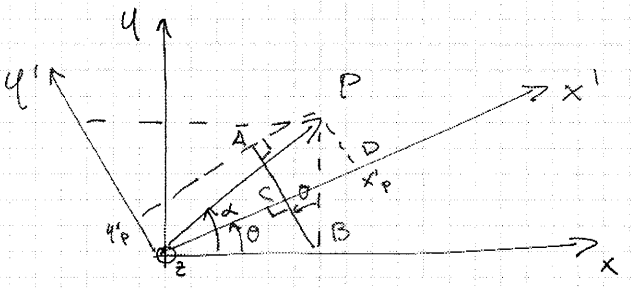
I vettori esprimono grandezze che non si possono esprimere solo con un numero, hanno bisogno di un verso ed una direzione per essere definiti. Ad esempio il vettore posizione specifica la posizione del p.to P (i corpi in prima approssimazione possono essere considerati p.ti materiali).



$$\overrightarrow{OP} = (x_P, y_P, z_P)$$

$$|\overrightarrow{OP}| = \sqrt{x_P^2 + y_P^2 + z_P^2}$$

Supponiamo di ruotare il sistema di assi attorno a z:



$$\begin{cases} x'_P = \overline{OC} + \overline{CD} = x_P \cos\theta + y_P \sin\theta \\ y'_P = \overline{AB} - \overline{CB} = y_P \cos\theta - x_P \sin\theta \end{cases}$$

MASTER COPY
Tel. 050 8312126
Cell. 388 9837745

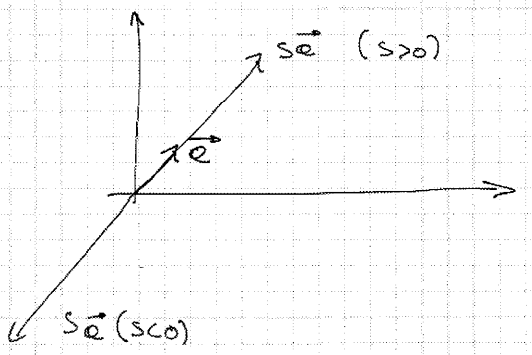
Operazioni

(i) MOLTIPLICAZIONE PER SCALARE

$$s\vec{a} = (sa_x, sa_y, sa_z)$$

$$|s\vec{a}| = \sqrt{(sa_x)^2 + (sa_y)^2 + (sa_z)^2} =$$

$$= |s| \sqrt{a_x^2 + a_y^2 + a_z^2} = |s|a$$



DEF: VERSORE (RIFERITO A UN VETTORE):

$\vec{n}$ versore $\quad$ $\vec{a}$ vettore

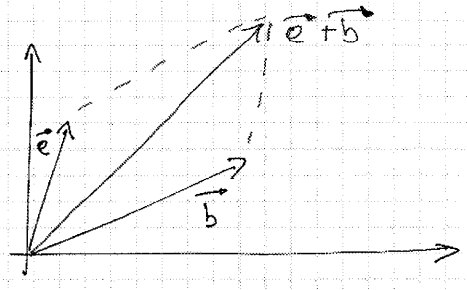
$$\vec{n} = \frac{\vec{a}}{a} = s\vec{a} \qquad s = \frac{1}{a}$$

$\vec{n}$ è adimensionale !

Nella terna cartesiana:

$\vec{i} = (1,0,0) \quad \vec{j} = (0,1,0) \quad \vec{k} = (0,0,1)$

(ii) SOMMA DI VETTORI



$$\vec{a} + \vec{b} = (a_x + b_x, a_y + b_y, a_z + b_z)$$

$$\vec{a} - \vec{b} = \vec{a} + (-1)\vec{b}$$

$\vec{v} = s_1\vec{a} + s_2\vec{b}$ comb. lineare di $\vec{a}$ e $\vec{b}$

qualsiasi vettore è comb. lineare di $\vec{i}, \vec{j}, \vec{k}$

$$\vec{a} = a_x\vec{i} + a_y\vec{j} + a_z\vec{k}$$

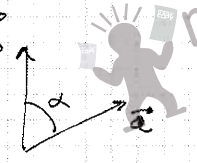
MASTER COPY
Tel. 050 8312126
Cell. 388 9837745

Proprietà:

① commutativa $\quad \vec{a}+\vec{b} = \vec{b}+\vec{a}$

② distributive $\quad s(\vec{a}+\vec{b}) = s\vec{a}+s\vec{b}$

③ associative $\quad (\vec{a}+\vec{b})+\vec{c} = \vec{a}+(\vec{b}+\vec{c})$

DEF: 

$\alpha \rightarrow$ angolo compreso tra $\vec{a}$ e $\vec{b}$

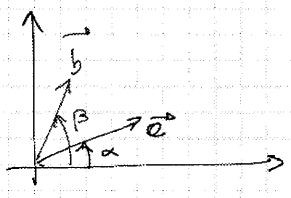
$\alpha \in [0, \pi]$

master COPY

(iii) PRODOTTO SCALARE

$$\vec{a}\cdot\vec{b} = a_x b_x + a_y b_y + a_z b_z = a\cos\alpha\, b\cos\alpha + a\sin\alpha\, b\cos\alpha =$$

$$= ab\cos(\alpha-\beta)$$



$\downarrow$

$ab\cos\varphi \rightarrow \varphi \rightarrow$ angolo tra $\vec{a}$ e $\vec{b}$

N.B. $\quad \vec{a}\cdot\vec{b} = 0 \Rightarrow \cos\varphi = 0 \Rightarrow \vec{a} \perp \vec{b}$

$$\vec{i}\cdot\vec{i} = 1 = \vec{j}\cdot\vec{j} = \vec{k}\cdot\vec{k}$$

MASTER COPY
Tel. 050 8312126
Cell. 388 9837745

Proprietà

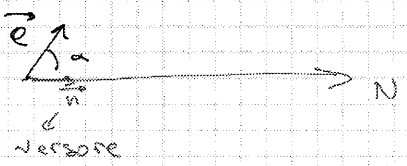
① commutative $\quad \vec{a}\cdot\vec{b} = \vec{b}\cdot\vec{a}$

② "associative" $\quad s(\vec{a}\cdot\vec{b}) = s\vec{a}\cdot\vec{b}$

③ distributive $\quad \vec{a}\cdot(\vec{b}+\vec{c}) = \vec{a}\cdot\vec{b} + \vec{a}\cdot\vec{c}$

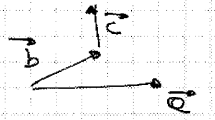
master COPY

DEF: COMPONENTE DI UN VETTORE LUNGO UNA DIREZIONE ORIENTATA



$$\vec{e}_N = \vec{e} \cdot \vec{n} = e \cos\alpha$$

N.B. esistono infiniti $\vec{b}'$ t.c. $\vec{b}' \cdot \vec{e} = \vec{b} \cdot \vec{e}$



$$\vec{b}' = \vec{b} + \vec{c}$$

$$\vec{e} \cdot (\vec{b} + \vec{c}) = \vec{e} \cdot \vec{b} + \vec{e} \cdot \vec{c}$$ ($\vec{e} \perp \vec{c}$)

(iiii) PRODOTTO VETTORIALE

$$\vec{e} \wedge \vec{b} = \begin{vmatrix} \vec{i} & \vec{j} & \vec{k} \\ e_x & e_y & e_z \\ b_x & b_y & b_z \end{vmatrix} = (e_y b_z - e_z b_y)\vec{i} + (e_z b_x - e_x b_z)\vec{j} + (e_x b_y - e_y b_x)\vec{k}$$

$$\begin{vmatrix} e_1 & e_2 & e_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix} = e_1 \begin{vmatrix} b_2 & b_3 \\ c_2 & c_3 \end{vmatrix} - e_2 \begin{vmatrix} b_1 & b_3 \\ c_1 & c_3 \end{vmatrix} + e_3 \begin{vmatrix} b_1 & b_2 \\ c_1 & c_2 \end{vmatrix}$$

$\vec{e} \wedge \vec{b} = 0 \Rightarrow \vec{e} \parallel \vec{b}$

$|\vec{e} \wedge \vec{b}| = eb \sin\varphi$ direzione → meno dx

Proprietà

1. anticommutative $\quad \vec{e} \wedge \vec{b} = -\vec{b} \wedge \vec{e}$
2. "associative" $\quad s(\vec{e} \wedge \vec{b}) = s\vec{e} \wedge \vec{b}$
3. distributive $\quad \vec{c} \wedge (\vec{e} + \vec{b}) = \vec{c} \wedge \vec{e} + \vec{c} \wedge \vec{b}$

MASTER COPY
Tel. 050 8312126
Cell. 388 9837745

N.B. esistono infiniti $\vec{b}'$ t.c. $\vec{b}' \wedge \vec{a} = \vec{b} \wedge \vec{a}$

$$\vec{b}' = s\vec{a} + \vec{b}$$

$$\vec{a} \wedge (s\vec{a} + \vec{b}) = \cancel{(\vec{a} \wedge s\vec{a})} + (\vec{a} \wedge \vec{b}) \qquad \vec{a} \parallel s\vec{a}$$

master COPY

Derivate

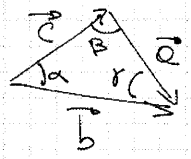
$$\frac{d}{dt}(\vec{A} \cdot \vec{B}) = \dot{\vec{A}} \cdot \vec{B} + \vec{A} \cdot \dot{\vec{B}}$$

$$\frac{d}{dt}(\vec{A} \wedge \vec{B}) = \dot{\vec{A}} \wedge \vec{B} + \vec{A} \wedge \dot{\vec{B}}$$

Teoremi

① CARNOT $\quad \vec{c} = \vec{a} + \vec{b} \Rightarrow \vec{c}^{\,2} = \vec{a}^{\,2} + \vec{b}^{\,2} + 2\,\vec{a} \cdot \vec{b}$

② SENI



$$\frac{a}{\sin\alpha} = \frac{b}{\sin\beta} = \frac{c}{\sin\gamma}$$

③ $\quad \vec{a} \wedge (\vec{b} \wedge \vec{c}) = (\vec{a} \cdot \vec{c})\,\vec{b} - (\vec{a} \cdot \vec{b})\,\vec{c}$

$$(\vec{a} \wedge \vec{b}) \wedge \vec{c} = (\vec{a} \cdot \vec{c})\,\vec{b} - (\vec{b} \cdot \vec{c})\,\vec{a}$$

master COPY

④ $\quad \vec{a} \cdot (\vec{b} \wedge \vec{c}) = \vec{b} \cdot (\vec{c} \wedge \vec{a}) = \vec{c} \cdot (\vec{a} \wedge \vec{b})$