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Theory of production and costs - Demand elasticity

The demand curve provides the demanded quantity when price varies

- Demand law: the demand curve has a negative slope function, the higher the price the lower the quantity. Company that wants to sell, lower the price and this relation is stable.
  - Direct demand curve: given the price, how many units are sold at that price?
  - Inverse demand curve: given the quantity, what is the price at which that quantity is sold? We can have also the inverse function.

Key issue: which is the % decreases in demand when a % increase in price occurs?
It depends on elasticity; we measure the relation in terms of quantity when we change the price.
If the decrease in demand is (very) limited, an increase in price leads to an increase in firm's revenues.
The intensity of the relation is measured by elasticity. At the limit they have perfectly substitutable products and no strategy, the more differentiated is the project, better will be the strategy.
Demand elasticity: relative variation of quantity (q) (or variation in %)/ relative variation of price (p) (or the variation in %).

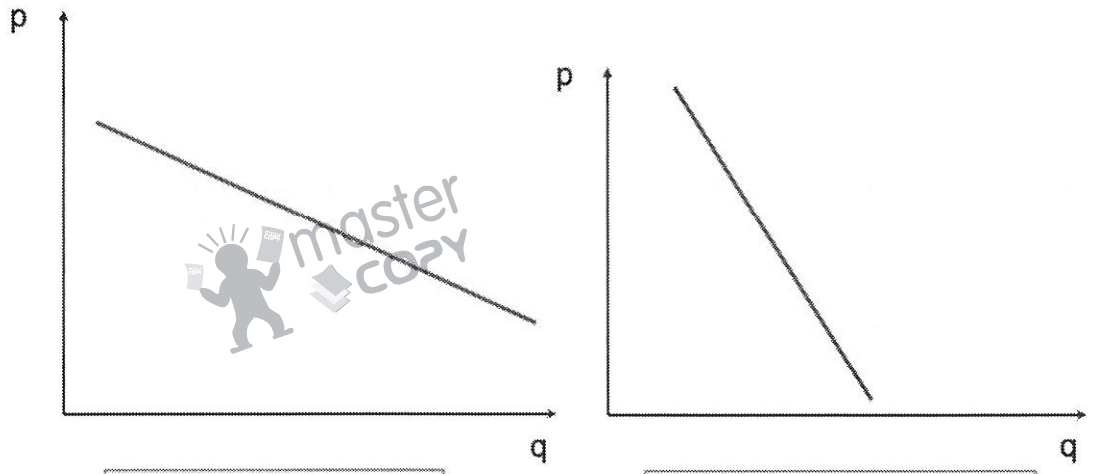
$$\varepsilon = \frac{\frac{\Delta q}{q}}{\frac{\Delta p}{p}} \Rightarrow \frac{\Delta q}{\Delta p} \cdot \frac{p}{q} \Rightarrow \frac{\partial q}{\partial p} \cdot \frac{p}{q}$$

If we put the p and q to the limit, we obtain the derivative.
We change the formula in such a way to not have ratios of ratios but product of ratios.
A variation in price generates a variation in quantity.
The variations are in opposite direction so this quantity is negative, so we use the magnitude of the elasticity.

1. Unelastic (rigid) demand: $|\varepsilon| < 1$
   - Low demand elasticity: an increase in price causes a limited reduction of demand. The quantity will decrease in few percentages.
     In this case the best strategy is to try to increase the price as much as possible, so to reduce the quantity in a small variation.
   - Steep demand curve.
2. Elastic demand: $|\varepsilon| > 1$
   - High demand elasticity: an increase in price causes a remarkable reduction of demand.
     In this case we are in trouble, any change in price can vary the quantity a lot.
   - Flat demand curve

The demand is more rigid when there are no substitutes.
Ex. Particular drugs, or energy (in the long-term can be substituted but not in the short-term).
We may have also imperfect substitution if there are substitutes but not perfect substitutes, we have loss of utility.
We talk about **differentiation** when we have limits to substitute ability, moving from a product to another.
We have *real* differentiation if there are some features objectively differentiated or *perceived* differentiation, that is still real even if we don't have objective differentiation, it's intuitive or emotional.

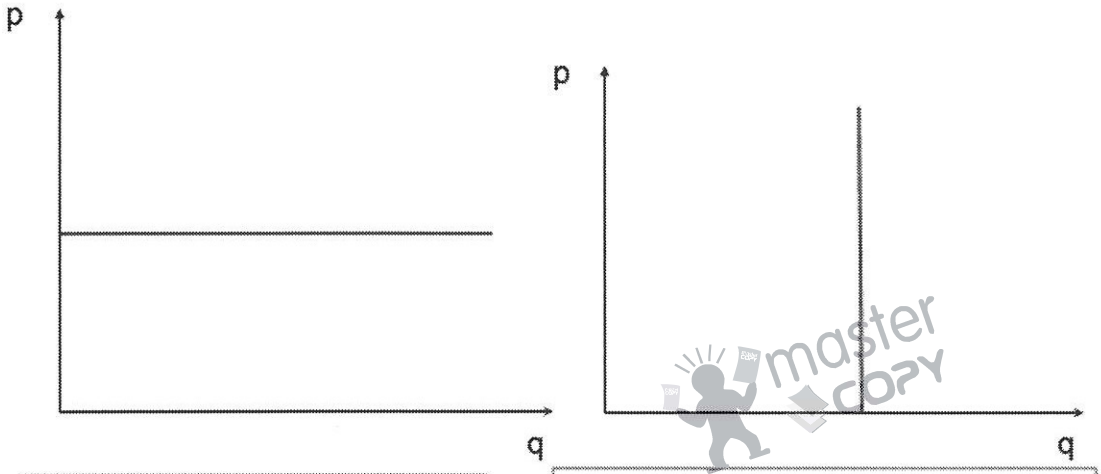
Ex. Driving experience with the excitement of going fast but with a more security, related to the emotional experience of people. The perception is real at the end.
Total revenue is the product between price and quantity, and it is influenced by elasticity.



Elastic demand curve

Unelastic demand curve

If we calculate the derivative, it is the inverse of the inclination, it is the angle.
In the second case it is lower than one.
In the first case the limit is a completely flat curve, present in perfect competition, even if we move a little bit the price we may go down, we have a perfect substitutability of products, products are homogeneous.
If the market is close to the competition, we must be careful.



Perfectly elastic demand curve: $|\varepsilon| \to \infty$

Perfectly unelastic demand curve $|\varepsilon| = 0$

If it's perfect horizontal the variation is such that variation make loss all the customers.
Vertical function means that even if we change the price, we don't lose any customer.
Ex. life-saving drugs

Product differentiation

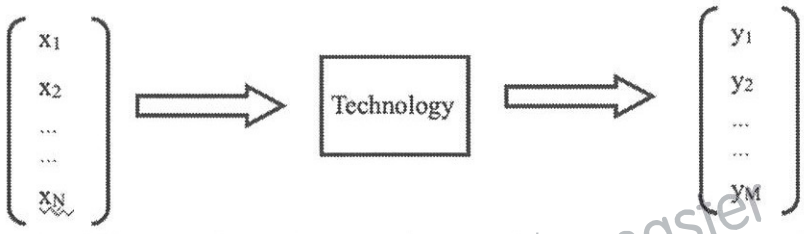
It will be important to reduce elasticity, because if there's no differentiation, there's no strategy.
If they are completely substitutable there is no strategy, they sell products at the price decided by the market and not their self. (price-taker, they take the price from the market)
If they increase the price they may lose all customers, if they reduce it they may lead other to do that too and also they cannot satisfy all the market.
Firms aim is to reduce the demand elasticity of their products so as increases in prices lead to increases in revenues.
It is possible to reduce the demand elasticity of a product by differentiating it from other products (of competitors and/or of the firm).
The brand is important, by using the brand we start to have differentiation.
By using the brand, you take the market power, the ability to charge the price higher than the price level at which all products are substitute.
The market power is the foundation of strategy.
Product differentiation is:

- Conducive to market power: it allows setting high prices without losing customers
- A widespread competitive strategy
  - Real differentiation: by changing the real characteristics of the product, for instance through innovation
  - Perceived differentiation: by changing consumers' perception of the characteristics of the product, for instance through advertising

Technology

The other side of the problem is the technology, which gives also other constraints.
When choosing quantities to produce and prices to charge, firms face constraints.

- Customers (demand), competitors (market structure), nature of the production process (technology)
- Technology: the set of processes that a firm can use to turn a vector X of N inputs into a vector Y of M outputs



We can see it in a simplified way in such a way that the input are just two, typically capital (equipment, machinery, all technical elements of the production process usable more than once) and labour (the personal) and the output is just one.
In a general way we can see it in a transformation from a vector to another vector and we don't care about the details of the transformation, with very abstract notions of it.

- Production set: the set of all combinations of inputs and outputs that comprise a technologically feasible way to produce.
- Production function: the maximum possible output from a given vector of inputs. It is the boundary of the production set. It answers the question: Given we have these machines and workers, what is the maximum production we can achieve? We cannot produce more than the production function; this is the maximum. If we produce less we are not dealing with it efficiently.
  Software is the only technology in which this may not fit.

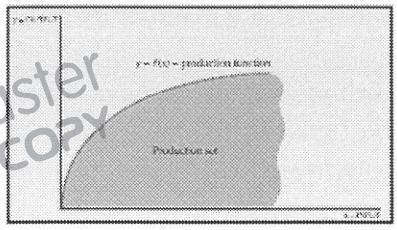
We design a relation in which we plot the level of production as a function of the level of input.
If we have N-1 input constants we can study the variation of the output in front of the variation of any input.

1. If there is 1 input x and 1 output y, the production function is **y=f(x)**

2. With 2 inputs, x_1 and x_2, and 1 output y, the production function is **$y=f(x_1, x_2)$**

Isoquant: set of all input bundles that produce the same output *y*.

We calculate where the variability of production is, when given the machine we change the hours of workers.



An isoquant captures this relation by studying all combinations between two inputs that generates that level of output.

We can combine different machines and people combining different level of production.

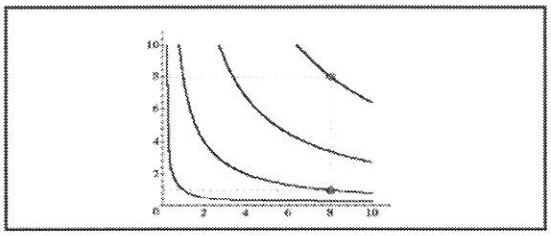
We fix the production to a number and then we describe all the combination of inputs that can generate this.

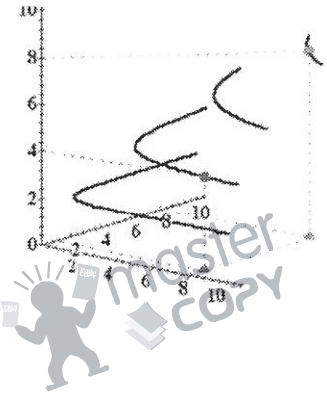
We combine differently machines and people fixed the level of production.

They are negatively shaped curves because we assume that each input has a positive partial derivative, contributes something in the production. If we add a machine this increase the production, if this is true, more machines will determine less people. If we increase one input then we need to decrease the other, and therefore it is negatively shaped.

It looks hyperbolic because it depends on the production function, which is a multiplicative function.

Isoquants can be graphed both in a x_1,x_2 plan and in 3D.





Relevant Production Functions

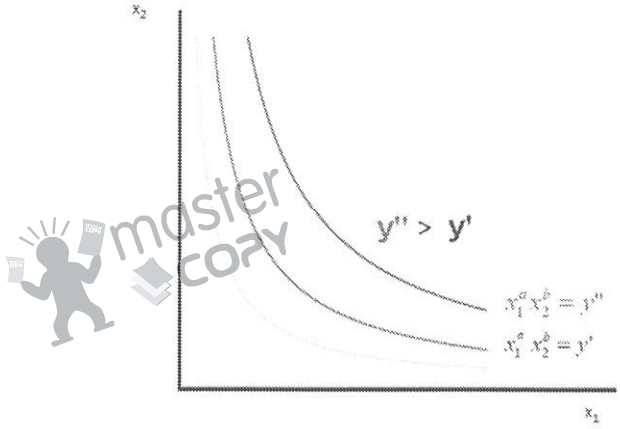
Cobb-Douglas Function

$$y = f(x_1, x_2) = Ax_1^a x_2^b$$

where x1 will be the capital and x2 the labor, for example.

- *A* measures the **scale of production**, i.e., the output obtained by using 1 unit of each input
  It is a scalar that gives the production depending on the input.
  - In the simplest case, A=1
- *a* and *b* measure how the output responds to changes in inputs
- Many production processes are described by Cobb-Douglas functions

We have a multiplicative function. If one is 0 it all will be 0. This means we need both capital and labor.
The assumption is that the marginal product is greater than 0, otherwise we have that more work gives less production.
We derive our isoquants and given that we don't want the two factors to be 0, it is asymptotic.



Isoquants are hyperbolic, asymptotic to the two axes.
If we consider the shape of this relation, the derivative, the tangent, is not constant, it is variable.
If we use more workers and capital, we go outside from the origin, the more we produce.
The inclination is the **marginal rate of substitution** (transformation), it tells us that we can substitute labor and capital to maintain the same level of production.
'How many hours of extra work we need for a man to substitute a machine to maintain the same level of production?' or the opposite.

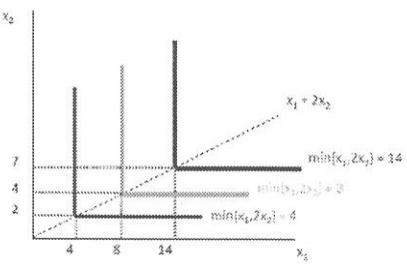
Leontief Fuction
This rate can be thought to be as constant, using the Leontief function.
In some cases, we have fixed proportions, we can produce only by combing capital and labor in the same proportion.

$$y = f(x_1, x_2) = \min\{x_1, x_2\}$$

To produce the output *y*, inputs must be used in fixed proportions.
There's no substitutability between inputs.
The more we increase the input the more we increase the output in cobb-douglass, here if we increase one of the input and not the other input, the increase of production is not present.
This is a production function in which the increase of product if we increase just one input is 0.
We take the derivative and call it marginal product to obtain an increase of production, which is how does the production increase when we add an additional unit of input.
In cobb-dauglas it is always positive, while in Leontief is 0.
Example: to produce 4 units of output, the firm needs at least 4 units of input 1 and 2 units of input 2

$$y = \min\{x_1, 2x_2\}$$

The two isoquants are with a 90° angle, if we move just one input we move horizontally or vertically but we do not increase production.
It is inefficient, we waste inputs, machines, or people.
We have inefficiency with Leontief, if we increase just one input we have waste. We need to find the perfect combination of them.



Isoquants are L-shaped