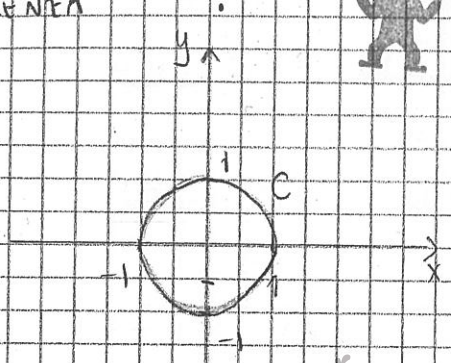
4/03/'20

• CIRCONFERENZA



$$C = \left\{ (x,y) \in \mathbb{R}^2 : \sqrt{x^2+y^2} = 1 \right\}$$

↓ circonferenza come sottoinsieme del piano

$$a = \left\{ (x,y) \in \mathbb{R}^2 : -1 \le x \le 1,\ y = \sqrt{1-x^2} \right\} \cup$$

$$\cup^{b} \left\{ (x,y) \in \mathbb{R}^2 : -1 \le x \le 1,\ y = \sqrt{1-x^2} \right\}$$

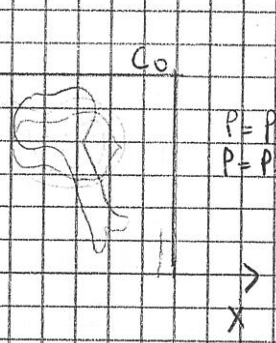
da circonferenza è un insieme? SI

• $f(x,y) = \sqrt{x^2+y^2}$ $\quad C = \left\{ (x,y) \in \mathbb{R}^2 : f(x,y) = 1 \right\}$

a $g_+ : [-1,1] \to \mathbb{R}$ $\quad g_+(x) = \sqrt{1-x^2}$ $\quad (\text{graf}\ g_+ = \left\{ (x,y) \in \mathbb{R}^2 : x \in [-1;1],\ y = g_+(x) \right\}$

b $g_- : [-1,1] \to \mathbb{R}$ $\quad g_-(x) = \sqrt{1+x^2}$ $\quad (\text{graf}\ g_- = \left\{ (x,y) \in \mathbb{R}^2 : x \in [-1;1],\ y = g_-(x) \right\}$

ES



$P(x,y)$ pressione in (x,y)

$C_0 = \left\{ (x,y) \in \mathbb{R}^2 : P(x,y) = P_0 \right\}$

$C_1 = \left\{ (x,y) \in \mathbb{R}^2 : P_1(x,y) = P_1 \right\}$

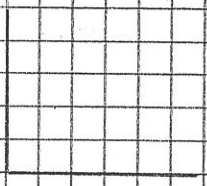
$\mathbb{R}^n$ → SPAZIO FISICO in cui vengono definiti i fenomeni

$P \in \mathbb{R}^n$, sistema di coordinate e origine O

$P = (P_1, P_2, \dots P_n)$ $\quad P_2 \in \mathbb{R}$

es.

$\mathbb{R}^2$ $\quad P(x,y)$



$\mathbb{R}^3$ $\quad P(x,y,z)$



Def. Si dice DISTANZA su $\mathbb{R}^n$ una funzione $d : \mathbb{R}^n \times \mathbb{R}^n \to \mathbb{R}$ t.c.

(i) POSITIVITÀ $\forall P, Q \in \mathbb{R}^n$ si ha $d(P,Q) \geq 0$ e

$$d(P,Q) = 0 \iff P = Q$$

(ii) SIMMETRIA $\forall P, Q \in \mathbb{R}^n$ si ha $d(P,Q) = d(Q,P)$

(iii) DISUGUAGLIANZA TRIANGOLARE $\forall P, Q, R \in \mathbb{R}^n$ si ha che

$$d(P,Q) \leq d(P,R) + d(R,Q)$$

es. $d(P,Q) \begin{cases} = 1 & P \neq Q \\ = 0 & P = Q \end{cases}$ (i)

(ii) $P \neq Q$, $Q \neq R$, $P \neq R$ $\quad 1 \leq 2$ $\qquad$ $P \neq Q$, $R = P$ $\quad 1 \leq 0+1$

i) verificata
ii) verificata
$P \neq Q$, $R = Q$ $\quad 1 \leq 1$

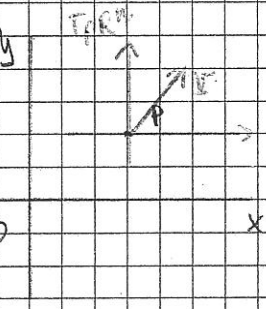
$\Downarrow$

iii) è verificata

DISTANZA EUCLIDEA $P = (p_1, \dots, p_n)$ $\quad Q = (q_1, \dots, q_n)$

$$d(P,Q) = \sqrt{(p_1 - q_1)^2 + \dots + (p_n - q_n)^2}$$

$\mathbb{R}^2$



Indichiamo con $T_P\mathbb{R}^n$ (spazio tangente a $\mathbb{R}^n$ in P) l'insieme dei vettori $\underline{v} = \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix}$ applicati in P. $T_P\mathbb{R}^n$ è uno spazio vettoriale isomorfo a $\mathbb{R}^n$

Def. Dato $T_P\mathbb{R}^n$ spazio vettoriale si definisce NORMA

$\|\cdot\| : T_P\mathbb{R}^n \to \mathbb{R}$ t.c.

(i) $\forall \underline{v} \in T_P\mathbb{R}^n$ si ha che $\|v\| \geq 0$ e $\|v\| = 0 \iff \underline{v} = 0 = \begin{pmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{pmatrix}$

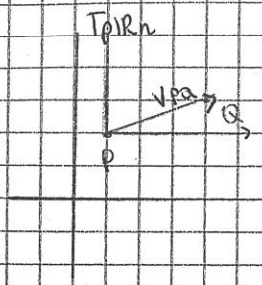
(ii) $\forall \underline{v} \in T_P\mathbb{R}^n$, $\forall t \in \mathbb{R}^n$, $\forall t \in \mathbb{R}$ si ha $\|t \cdot v\| = |t| \cdot \|v\|$

(iii) $\forall v, w \in T_P\mathbb{R}^n$ si ha $\|v + w\| \leq \|v\| + \|w\|$

Def. Norma euclidea. Data $v \in T_P\mathbb{R}^n$

$$v = \begin{pmatrix} v_1 \\ \vdots \\ v_n \end{pmatrix}, \quad \|v\| = \sqrt{v_1^2 + \dots + v_n^2}$$

$P, Q \in \mathbb{R}^n$ e se $T_P\mathbb{R}^n$ è dotato di norma $d(P,Q) = \|v_{PQ}\|$, $v_{PQ} \in T_P\mathbb{R}^n$

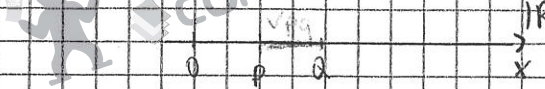


$P = (P_1 \dots P_n)$ $\quad Q = (Q_1 \dots Q_n)$

$$v_{PQ} = \begin{pmatrix} q_1 - p_1 \\ | \\ q_n - p_n \end{pmatrix}$$

Una norma introduce una distanza ma non vale il viceversa

Esempio: $(\mathbb{R}, d)$

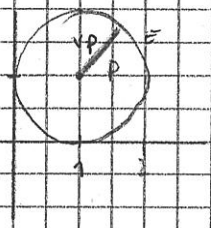


$P = (x)$, $Q = (x')$ $\quad d(P,Q) = \sqrt{(x-x')^2} = |x-x'|$

↓ distanza euclidea

$v \in T_P\mathbb{R}$ $\quad v_{PQ} = (x'-x)$, $\quad \|v_{PQ}\| = \sqrt{(x'-x)^2} = |x'-x|$

Esempio: $\mathbb{R}^2$ $\quad P = (1,1)$ $\quad E = \{ Q \in \mathbb{R}^2 : \|v_{PQ}\| = 1 \}$



$Q = (x,y) \in \mathbb{R}^2$

$v_{PQ} \in T_P\mathbb{R}^n$, $v_{PQ} = \begin{pmatrix} x-1 \\ y-1 \end{pmatrix}$

$\|v_P\| = 1 \iff \sqrt{(x-1)^2 + (y-1)^2} = 1$

$\iff (x-1)^2 + (y-1)^2 = 1$ CIRCONFERENZA centro C e raggio 1

$\iff d(P,Q) = 1$ distanza euclidea

SPAZIO METRICO

$(\mathbb{R}^n, d)$ $\quad$ **Def.** Dati $P \in \mathbb{R}^n$, $\varepsilon > 0$ si chiama INTORNO SFERICO DI P DI RAGGIO ε l'insieme $B_\varepsilon(P)$ dato da

$$B_\varepsilon(P) = \{ Q \in \mathbb{R}^n : d(P,Q) < \varepsilon \}$$

«Ball»

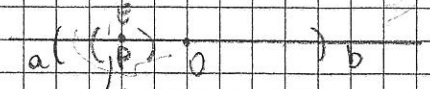
es $n=1$ $\quad B_\varepsilon(x) = (x-\varepsilon, x+\varepsilon)$

Def. Dato $E \in \mathbb{R}^n$ si dice che $P \in \mathbb{R}^n$ è di accumulazione per E se $\forall \varepsilon > 0$

$\exists Q \in E \cap B_\varepsilon(P) \setminus \{P\}$

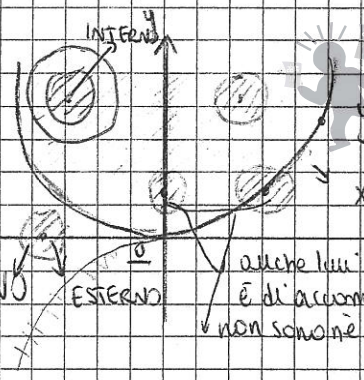
es. $E = (a,b) \in \mathbb{R}$ $\quad Acc(E) =$

(i non appartenenza non è necessariamente a E.)



Ci sono punti di acc. di E che non stanno in E? Sì
a è bordo dell'insieme perché qualsiasi intorno interseca E

es $E = \{(x,y) \in \mathbb{R}^2 : y \geq |x|^3 \ , \ x \neq 0\}$



$$Acc(E) = \{(x,y) \in \mathbb{R}^2 \ : \ y \geq |x|^3\}$$

Def. Dato $E \in \mathbb{R}^n$ si dice che $P \in E$ è ISOLATO se $\exists\, B_\varepsilon(P)$ t.c. $B_\varepsilon(P) \cap E = \{P\}$ (intersezione con)

Def. Dato $E \in \mathbb{R}^n$ si dice che $P \in E$ è INTERNO ad E se $\exists\, B_\varepsilon(P) : \subset E$

Parte interna di $E = \mathring{E} = \{$ punti interni di $E\}$

$$* \quad \mathring{E} = \{(x,y) \in \mathbb{R}^2 : (x,y) \in E \text{ t.c. } y > |x|^3\}$$

Def. Dato $E \in \mathbb{R}^n$, si dice che $P \in \mathbb{R}^n$ è ESTERNO ad E se

P è interno a $\mathbb{R}^n \setminus E$ $\quad (\Leftrightarrow \exists\, B_\varepsilon(P)$ t.c. $B_\varepsilon(P) \subset \mathbb{R}^n \setminus E)$

Si dice che $P \in \mathbb{R}^n$ è di FRONTIERA per E se P non è né interno né esterno a E

Frontiera di $E = \partial E = \{$ punti di frontiera $\}$
(Bordo)

$$\partial E = \{y = |x|^3\} \cup \{y \geq 0, \ x = 0\}$$

Def. $E \in \mathbb{R}^n$, chiusura di $E = \overline{E} = E \cup \partial E$

$E \subset \mathbb{R}^n$ si dice CHIUSO $E = \overline{E}$

$E \subset \mathbb{R}^n$ si dice APERTO $E = \mathring{E}$

$d(x,0) = \sqrt{(x-0)^2}$

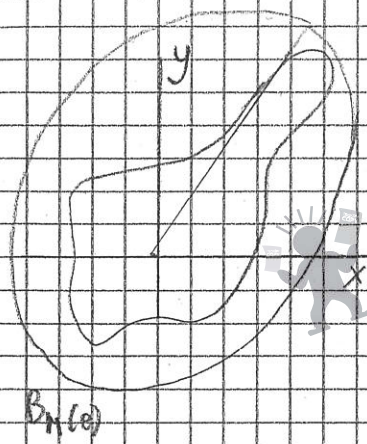
$E_0 = [0,3)$

$E = [0,+\infty)$

Def. $E \subset \mathbb{R}$ è LIMITATO se $\exists M > 0$ t.c. $|x| \leq M \quad \forall x \in E$

Def. $E \subset \mathbb{R}^n$ è LIMITATO se $\exists M > 0$ t.c. $d(P,Q) < M$ $\forall P \in E$

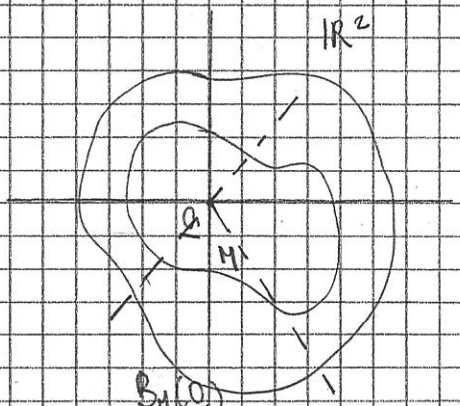
$(\Leftrightarrow E \subset B_M(Q))$



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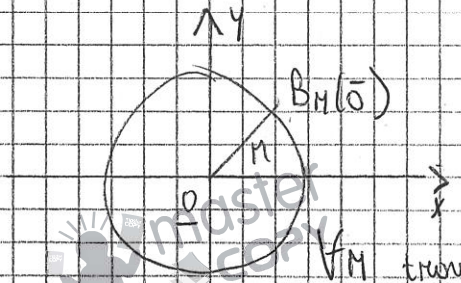
$E \subset \mathbb{R}^n$, $n \geq 1$

Def. E è <u>limitato</u> se $\exists M > 0$ t.c. $d(P,O) < M \quad \forall P \in E$



LIMITATO

$E = \{y = 0\} \subset \mathbb{R}^2$



$\forall M$ trovo punti che stanno fuori

NON LIMITATO