OPERAZIONI MATRICIALI (I)

$\underline{A}$ => MATRICE $\vec{N}$ => VETTORE

a_{ij} => COMPONENTE MATRICIALE N_i => COMPONENTE VETTORIALE

$$\underline{A} \cdot \underline{B} = \underline{C}$$
$$(m \times p) \cdot (p \times n) = (m \times n)$$
$$(3 \times 3) \cdot (3 \times 1) = (3 \times 1)$$



$$\underline{A} \cdot \vec{b} = \vec{c}$$

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \cdot \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix} = \begin{bmatrix} a_{11}b_1 + a_{12}b_2 + a_{13}b_3 \\ a_{21}b_1 + a_{22}b_2 + a_{23}b_3 \\ a_{31}b_1 + a_{32}b_2 + a_{33}b_3 \end{bmatrix}$$

$\underline{A} \cdot \vec{b} = \vec{c}$

$$c_i = \sum_{j=1}^{3} a_{ij} b_j$$

i = indice INTERO

j = INDICE MUTO

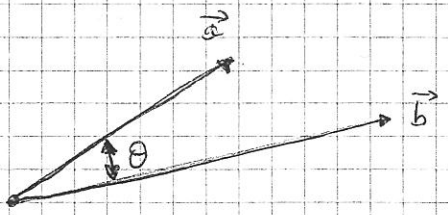
$\Downarrow$

$c_i = a_{ij} b_j$ => semplificazione di Einstein; si omette il simbolo di sommatoria dato che ci sono 2 indici j.

$(i,j = 1,2,3)$

• Prodotto scalare

$\vec{a} \cdot \vec{b} = |a||b| \cos\theta$

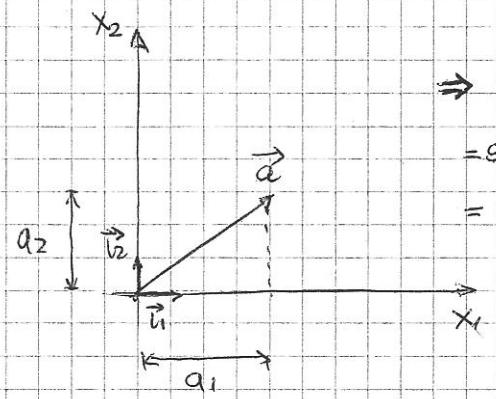


$\vec{a} \cdot \vec{b} = 0 \Rightarrow \vec{a} \perp \vec{b}$

Proprietà: 1) COMMUTATIVO $\vec{a} \cdot \vec{b} = \vec{b} \cdot \vec{a}$

2) DISTRIBUTIVO $\vec{a} \cdot (\vec{b} + \vec{c}) = \vec{a} \cdot \vec{b} + \vec{a} \cdot \vec{c}$

3) OMOGENEITÀ $\lambda(\vec{a} \cdot \vec{b}) = (\lambda \vec{a}) \cdot \vec{b} = \vec{a} \cdot (\lambda \vec{b})$



$\Rightarrow \vec{a} \cdot \vec{b} = (a_1 \vec{i}_1 + a_2 \vec{i}_2)(b_1 \vec{i}_1 + b_2 \vec{i}_2) =$

$= a_1 b_1 \vec{i}_1 \vec{i}_1 + a_1 b_2 \vec{i}_1 \vec{i}_2 + a_2 b_1 \vec{i}_2 \vec{i}_1 + a_2 b_2 \vec{i}_2 \vec{i}_2$

$= a_1 b_1 + a_2 b_2 = \sum_{i=1}^{2} a_i b_i = a_i b_i$

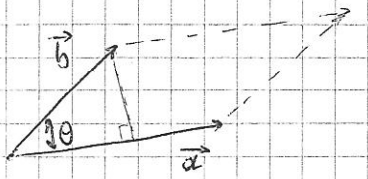
$\vec{a} = a_1 \vec{i}_1 + a_2 \vec{i}_2$

$\vec{b} = b_1 \vec{i}_1 + b_2 \vec{i}_2$

• Prodotto vettoriale

$\vec{a} \wedge \vec{b} = \vec{N}$

$|\vec{N}| = |a||b| \sin\theta$ $\Rightarrow$ il valore del modulo di $\vec{N}$ è pari all'area del parallelogramma individuato da $\vec{a}$ e $\vec{b}$



$\vec{a} \wedge \vec{b} = 0 \Rightarrow \vec{a} \parallel \vec{b}$

Proprietà: 1) ANTICOMMUTATIVO $\vec{a} \wedge \vec{b} = -\vec{b} \wedge \vec{a}$

2) DISTRIBUTIVO $\vec{a} \wedge (\vec{b} + \vec{c}) = \vec{a} \wedge \vec{b} + \vec{a} \wedge \vec{c}$

3) OMOGENEITÀ $\lambda(\vec{a} \wedge \vec{b}) = (\lambda \vec{a}) \wedge \vec{b} = \vec{a} \wedge (\lambda \vec{b})$

$$\vec{a} \wedge \vec{b} = (a_1 \vec{\imath}_1 + a_2 \vec{\imath}_2 + a_3 \vec{\imath}_3) \wedge (b_1 \vec{\imath}_1 + b_2 \vec{\imath}_2 + b_3 \vec{\imath}_3) =$$

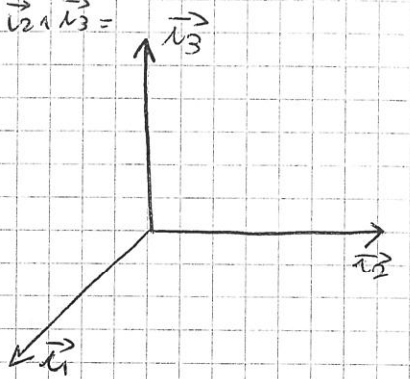
$$= a_1 b_1 \cancel{\vec{\imath}_1 \wedge \vec{\imath}_1} + a_1 b_2 \vec{\imath}_1 \wedge \vec{\imath}_2 + a_1 b_3 \vec{\imath}_1 \wedge \vec{\imath}_3 + a_2 b_1 \vec{\imath}_2 \wedge \vec{\imath}_1 + a_2 b_2 \cancel{\vec{\imath}_2 \wedge \vec{\imath}_2} +$$

$$+ a_3 b_1 \vec{\imath}_3 \wedge \vec{\imath}_1 + a_3 b_2 \vec{\imath}_3 \wedge \vec{\imath}_2 + a_3 b_3 \cancel{\vec{\imath}_3 \wedge \vec{\imath}_3} + a_2 b_3 \vec{\imath}_2 \wedge \vec{\imath}_3 =$$

$$= a_1 b_2 \vec{\imath}_3 + a_1 b_3 (-\vec{\imath}_2) + a_2 b_1 (-\vec{\imath}_3) +$$

$$+ a_3 b_1 (\vec{\imath}_2) + a_3 b_2 (-\vec{\imath}_1) + a_2 b_3 (\vec{\imath}_1) =$$

$$= (a_2 b_3 - a_3 b_2) \vec{\imath}_1 + (a_3 b_1 - a_1 b_3) \vec{\imath}_2 + (a_1 b_2 - a_2 b_1) \vec{\imath}_3$$



$$\vec{\imath}_1 \wedge \vec{\imath}_2 = \vec{\imath}_3$$
$$\vec{\imath}_1 \wedge \vec{\imath}_3 = -\vec{\imath}_2$$
$$\vec{\imath}_2 \wedge \vec{\imath}_3 = \vec{\imath}_1$$

Il risultato ottenuto si può calcolare anche tramite l'uso del determinante simbolico:

$$\Rightarrow \det \begin{vmatrix} \vec{\imath}_1 & \vec{\imath}_2 & \vec{\imath}_3 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = (a_2 b_3 - a_3 b_2) \vec{\imath}_1 + (a_3 b_1 - a_1 b_3) \vec{\imath}_2 + (a_1 b_2 - a_2 b_1) \vec{\imath}_3$$

In forma più completa possiamo scrivere:

$$\vec{a} \wedge \vec{b} = \vec{w} \quad \Rightarrow w_i = \sum_{j,k=1,3} e_{ijk} a_j b_k = \underset{i,j,k}{e} a_j b_k$$

$$e_{ijk} = \begin{cases} +1 & \text{se } i,j,k = 1,2,3 \text{ o } i,j,k = 3,1,2 \text{ o } i,j,k = 2,3,1 \quad \text{PERMUTAZIONI PARI} \\ -1 & \text{se } i,j,k = 3,2,1 \text{ o } i,j,k = 2,1,3 \text{ o } i,j,k = 1,3,2 \quad \text{PERMUTAZIONI DISPARI} \\ 0 & \text{se } i = j \text{ o } j = k \text{ o } i = k \end{cases}$$

SIMBOLO DI PERMUTAZIONE

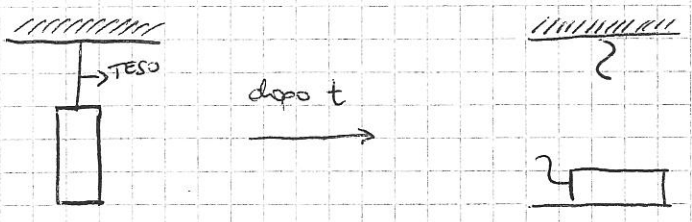
SIMBOLO DI LEVI-CIVITÀ

SIMBOLO DI RICCI

TENSORE DI LEVI-CIVITÀ

master COPY

STABILITÀ E STATICA (B)

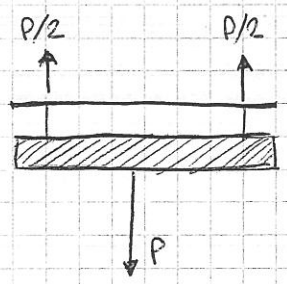


Posso agire in diversi modi:

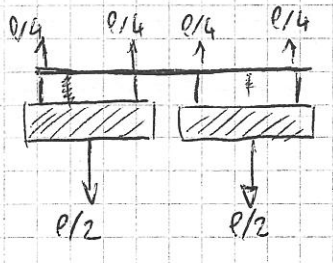
1) FILO PIÙ SPESSO

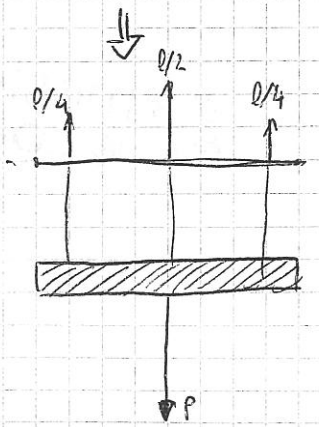
2) FILO PIÙ RESISTENTE

3) UTILIZZO DI PIÙ FILI, SPECIFICANDO LE POSIZIONI

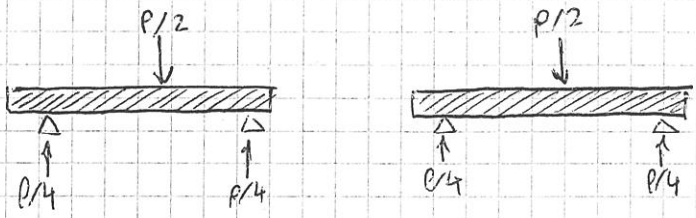


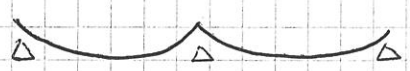
Posso usare una trave che pesa la metà:





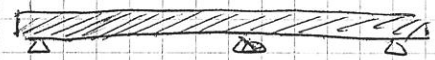
a)

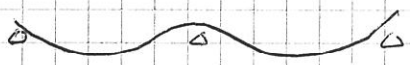




Se avviciniamo ed uniamo le 2 travi si ottiene un oggetto con una deformazione ellittica

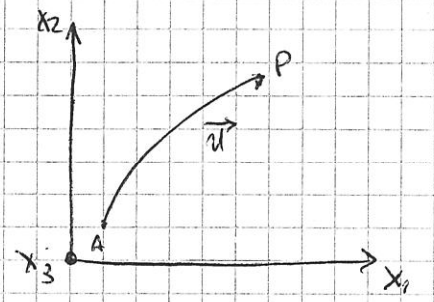
b)





In questo caso invece si ha una deformazione più lunga rispetto al problema precedente

Una forza agisce su di una materia se la mette in moto o riesce a deformarla.



$(A, \vec{u})$ o $(P-A)$ $\Rightarrow$ descrizione del vettore

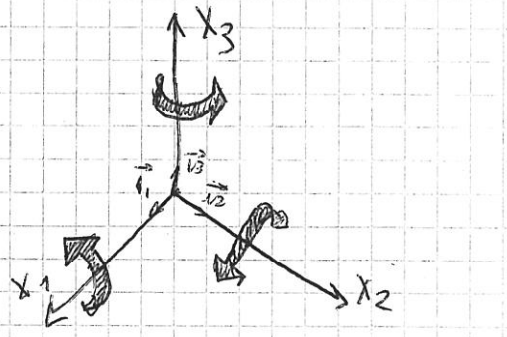
Una rotazione è positiva se è concorde con la TERNA LEVOGIRA

$$A: \begin{cases} x_1(A) \\ x_2(A) \\ x_3(A) \end{cases} \qquad \vec{u} \begin{cases} u_1 \\ u_2 \\ u_3 \end{cases} \qquad \vec{u} = u_1\vec{\imath}_1 + u_2\vec{\imath}_2 + u_3\vec{\imath}_3$$

(versori della terna unitaria)

$$P-A = \begin{cases} x_1(P) - x_1(A) \\ x_2(P) - x_2(A) \\ x_3(P) - x_3(A) \end{cases}$$

$$u_i = x_i(P) - x_i(A) \quad i = 1,2,3$$



x_1, x_2, x_3 => TERNA LEVOGIRA,

x_1 si sovrappone ad x_2 ruotato in senso antiorario.