Briciole di Analisi Matematica 1

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Formule Trigonometriche

Formule di Addizione

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta \qquad \cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

Formule di Duplicazione

$$\sin(2x) = 2\sin x\cos x \qquad \cos(2x) = \cos^2 x - \sin^2 x = 1 - 2\sin^2 x = 2\cos^2 x - 1$$

$$\sin^2 x = \frac{1-\cos 2x}{2} \qquad \cos^2 x = \frac{1+\cos 2x}{2}$$

Formule di Prostaferesi

$$\sin\alpha \pm \sin\beta = 2\sin\left(\frac{\alpha \pm \beta}{2}\right)\cos(\frac{\alpha \mp \beta}{2}) \qquad \cos\alpha + \cos\beta = 2\cos\left(\frac{\alpha+\beta}{2}\right)\cos\left(\frac{\alpha-\beta}{2}\right)$$

$$\cos\alpha - \cos\beta = -2\sin\left(\frac{\alpha+\beta}{2}\right)\sin\left(\frac{\alpha-\beta}{2}\right)$$

Formule di Werner

$$\sin\alpha\sin\beta = \frac{\cos(\alpha-\beta) - \cos(\alpha+\beta)}{2} \qquad \sin\alpha\cos\beta = \frac{\sin(\alpha+\beta) + \sin(\alpha-\beta)}{2}$$

$$\cos\alpha\cos\beta = \frac{\cos(\alpha+\beta) + \cos(\alpha-\beta)}{2} \qquad \sin\alpha\cos\alpha = \frac{\sin 2\alpha}{2}$$

Formule di Bisezione

$$\sin\frac{x}{2} = \sqrt{\frac{1-\cos x}{2}} \qquad \cos\frac{x}{2} = \sqrt{\frac{1+\cos x}{2}}$$

$$\tan\frac{x}{2} = \sqrt{\frac{1-\cos x}{1+\cos x}} = \frac{1-\cos x}{\sin x} = \frac{\sin x}{1+\cos x}$$

Somma e Sottrazione di Tangenti

$$\tan\alpha + \tan\beta = \frac{\sin(\alpha+\beta)}{\cos\alpha\cos\beta} \qquad \tan\alpha - \tan\beta = \frac{\sin(\alpha-\beta)}{\cos\alpha\cos\beta}$$

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Formule Parametriche

$$\text{Posto } t = \tan\left(\frac{x}{2}\right) \quad \text{si ha :}$$

$$\sin x = \frac{2t}{1+t^2} \qquad \cos x = \frac{1-t^2}{1+t^2}$$

$$\tan x = \frac{2t}{1-t^2}$$

Numeri Complessi

Potenze di i

$$\begin{array}{llll} i^1 = i & i^5 = i^4 i = i & i^9 = i^8 i = i & \longrightarrow \quad i^{4n+1} = i \\ i^2 = ii = -1 & i^6 = i^5 i = -1 & i^{10} = i^9 i = -1 & \longrightarrow \quad i^{4n+2} = -1 \\ i^3 = i^2 i = -i & i^7 = i^6 i = -i & i^{11} = i^{10} i = -i & \longrightarrow \quad i^{4n+3} = -i \\ i^4 = i^3 i = 1 & i^8 = i^7 i = 1 & i^{12} = i^{11} i = 1 & \longrightarrow \quad i^{4n} = 1 \end{array}$$

Proprietà Numeri Complessi

$$\overline{z+w} = \overline{z} + \overline{w} \qquad \overline{zw} = \overline{z}\,\overline{w}$$

$$z + \overline{z} = 2\mathbb{R}(z) \qquad \mathbb{R}(z+w) = \mathbb{R}(z) + \mathbb{R}(w)$$

$$\mathbb{I}(z+w) = \mathbb{I}(z) + \mathbb{I}(w) \qquad z - \overline{z} = 2\mathbb{I}(z)$$

$$z\overline{z} = (\mathbb{R}(z) + \mathbb{I}(z))^2 \qquad \overline{\overline{z}} = z$$

$$\mathbb{R}(z) = \frac{z+\overline{z}}{2} \qquad |\, z \,|^2 = z\overline{z}$$

$$|\, zw \,| = |\, z \,|\, |\, w \,| \qquad \overline{\left(\frac{z}{w}\right)} = \frac{\overline{z}}{\overline{w}}$$

$$|\, z \,| \geqslant 0 \qquad |\, z \,| = 0 \Leftrightarrow z = 0$$

$$\left|\, \frac{z}{w} \,\right| = \frac{|\, z \,|}{|\, w \,|} \; w \neq 0 \qquad |\, \mathbb{R}(z) \,| \leqslant |\, z \,|$$

$$|\, \mathbb{I}(z) \,| \leqslant |\, z \,| \qquad |\, z \,| \leqslant |\, \mathbb{R}(z) \,| + |\, \mathbb{I}(z)$$

$$|\, z + w \,| \leqslant |\, z \,| + |\, w \,| \qquad ||\, z \,| - |\, w \,|| \leqslant |\, z + w \,|$$

$$\cos(z) = \frac{e^{iz} + e^{-iz}}{2} \qquad \sin(z) = \frac{e^{iz} - e^{-iz}}{2i}$$

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Limiti

Limiti di successioni Notevoli

$$\lim_n n^a = +\infty \qquad \forall a > 0$$

$$\lim_n n^a = 0^+ \qquad \forall a < 0$$

$$\lim_n a^n = +\infty \qquad \forall a > 1$$

$$\lim_n a^n = 0 \qquad \forall a \in (0,1)$$

$$\lim_n n! = +\infty$$

$$\lim_n \sqrt[n]{a} = 1 \qquad \forall a > 0$$

$$\lim_n \frac{a^n}{n^b} = +\infty \qquad \forall a > 1 \qquad \forall b > 0$$

$$\lim_n \frac{a^n}{n!} = 0 \qquad \forall a > 1$$

$$\lim_n \left(1 + \frac{1}{n}\right)^n = e = 2,71...$$

$$\lim_n \frac{n^n}{n!} = +\infty$$

$$\lim_n \sqrt[n]{\text{polinomio}} = 1 \qquad \forall\, polinomio$$

$$\lim_n \sqrt[n]{n!} = +\infty$$

$$\lim_n \frac{\sqrt[n]{n!}}{n} \to \frac{1}{e}$$

$$\lim_n \frac{n^\alpha}{e^n} = 0^+ \qquad \forall \alpha \in \mathbb{R}$$

$$\lim_n \frac{log(n)}{e^n} = 0 \qquad \binom{n}{k} = \frac{n!}{(n-k)!k!}$$

$$\lim_n \frac{log(n)}{n!} = 0$$

$$\lim_n \frac{log(n)}{n^n} = 0$$

$$\lim_n \frac{e^n}{n!} = 0$$

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Limiti di funzioni Notevoli

$$\lim_{x\to+\infty}(1+\frac{a}{x})^{nx}=e^{na} \qquad \lim_{x\to0}\frac{1-\cos x}{x}=0$$

$$\lim_{x\to0}\frac{e^x-1}{x}=1 \qquad \lim_{x\to+\infty}(1+\frac{a}{x})^{x}=e^{a}$$

$$\lim_{x\to0}\frac{\log(1+x)}{x}=1 \qquad \lim_{x\to0}\frac{\log_a(1+x)}{x}=\frac{1}{\log a}$$

$$\lim_{x\to0}\frac{e^x-1}{x}=1 \qquad \lim_{x\to0}\frac{a^x-1}{x}=\log a$$

$$\lim_{x\to\pm\infty}(1+\frac{1}{x})^{x}=e \qquad \lim_{x\to0}\frac{(1+x)^c-1}{x}=c$$

$$\lim_{x\to0}\frac{\sin(x)}{x}=1 \qquad \lim_{x\to0}\frac{\sin ax}{bx}=\frac{a}{b}$$

$$\lim_{x\to0}\frac{1-\cos(x)}{x^2}=\frac{1}{2} \qquad \lim_{x\to0}\frac{\tan(x)}{x}=1$$

$$\lim_{x\to0}\frac{\tan ax}{bx}=\frac{a}{b} \qquad \lim_{x\to0}\frac{\arcsin(x)}{x}=1$$

$$\lim_{x\to0}\frac{\arcsin ax}{bx}=\frac{a}{b} \qquad \lim_{x\to0}\frac{\arctan(x)}{x}=1$$

$$\lim_{x\to0}\frac{\arctan ax}{bx}=\frac{a}{b} \qquad \lim_{x\to0}\frac{1-\cos x}{x}=0$$

$$\lim_{x\to0}\frac{x-\sin x}{x^3}=\frac{1}{6} \qquad \lim_{x\to0}\frac{x-\arctan x}{x^3}=\frac{1}{3}$$

$$\lim_{x\to0}(1+x)^{\frac{1}{x}}=e \qquad \lim_{x\to0}\frac{1-\cos x}{x^2}=\frac{1}{2}$$