PROVE D'ESAME SVOLTE DAL 2008 AL 2017 ED ESERCITAZIONI PROF. PALAIA

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(11)

- $\sum_{n\geq 1} \frac{(2n)!}{n^n \cdot n!} \to \lim_{n\to+\infty} \frac{(2n+2)(2n+1)(2n)!}{(n+1)^n (n+1)(n!)} \cdot \frac{n^n \cdot n!}{(2n)!} = 4$

diverge

- $\sum_{n\geq 1} \frac{n^n \cdot n!}{(2n)!} \to 4^{-1}$ converge

- $\sum_{n\geq 1} \frac{(2n)!}{(n!)^3} \to \lim_{n\to+\infty} \frac{(2n+2)(2n+1)(2n)!}{(n+1)^3 (n!)^3} \to 0 \cdot \frac{(n!)^3}{(2n)!}$ converge

(13) $y' + y' \operatorname{tg} x = \cos x \qquad y(0) = 1$

$a = \operatorname{tg} x \to \int \frac{\sin x}{\cos x} dx = -\ln|\cos x|$

$y = |\cos x| \left(\int \frac{\cos x}{|\cos x|} dx + C \right) \to y = \cos x (x + C)$

$1 = C$

$y = x \cos x + C \cos x$

(14)

$y'' - 2y' + y = x^2 e^x \to$ ecu. $(z-1)^2 \to z = 1 \to y_{om} = C_1 e^x + C_2 e^x x$

$f(x): a = 1$ dublu $\quad P(x) = Ax^2 + Bx + C \quad y^* = e^x (Ax^4 + Bx^3 + Cx^2)$

$y' = e^x (Ax^4 + Bx^3 + Cx^2) + e^x (4Ax^3 + 3Bx^2 + 2Cx)$

$y'' = e^x (Ax^4 + Bx^3 + Cx^2) + e^x (4Ax^3 + 3Bx^2 + 2Cx) + e^x (4Ax^3 + 3Bx^2 + 2Cx) + e^x (12Ax^2 + 6Bx + 2C)$

$\to Ax^4 + x^3(B + 4A + 4A) + x^2(C + 3B + 3B + 12A) + x(2C + 6B) + 2C - 2Ax^4 - 2Bx^3 - 2Cx^2 - 8Ax^3 - 6Bx^2 - 4Cx + Ax^4 + Bx^3 + Cx^2 = x^2$

$A = \frac{1}{12} \qquad B = 0 \qquad C = 0$

$\to y = C_1 e^x + C_2 x e^x + \frac{x^4}{12} e^x$

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① $|z-i| \leq |z+i|$

$|a+i(b-1)| \leq |a+i(b+1)|$

$a^2+(b-1)^2 \leq a^2+(b+1)^2 \Rightarrow$ Sol: $z = a+ib$ con $b \geq 0$

$b^2-2b+1 \leq b^2+2b+1$

$4b \geq 0$

$b \geq 0 \ \forall a$

② $a, b: z^2+aiz+b=0$

almeno 1 sol. compresa con $Re=0$

$2z = -ai \pm \sqrt{-a^2-4b}$

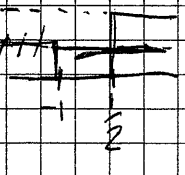
$2z = -ai \pm \sqrt{-a^2-4b} \rightarrow Re=0 \Rightarrow -a^2-4b=0$ $\wedge a \neq 0$

$b = -\frac{a^2}{4}$

oppure $a^2+4b \geq 0$ $b \geq -\frac{a^2}{4}$ → raccolgo anche il precedente

③ $f(x) = \sqrt{|x+1|-|2x+1|}$ CdE: $|x+1| > |2x+1|$ ① ② ③

$x > -1$ ↙ ↳ $x > \frac{1}{2}$

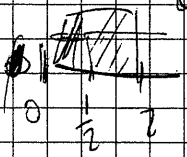


- $x < -1$

$-(x+1) > (2x+1)$ (1) $\Rightarrow x+1 \leq 2x-1$ $x \geq 2$ $\emptyset$

- $-1 \leq x \leq \frac{1}{2} \rightarrow -2x+1 < x+1$ $3x > 0$ $x > 0$ $\rightarrow 0 < x \leq \frac{1}{2}$

- $x \geq \frac{1}{2} \Rightarrow x+1 > 2x-1$ $x \leq 2$



$\Rightarrow 0 \leq x \leq 2$

④ $\lim_{n\to+\infty} n \sin\left(\frac{n^2(n-1)}{\mu} - 3\pi\right)$ $\quad \mu \in \mathbb{N}$

$\sin\left(n(n-1)\pi - \frac{3}{\mu}\pi\right) = \sin(n(n-1)\pi)\cos\left(\frac{3\pi}{\mu}\right) - \sin\left(\frac{3\pi}{\mu}\right)\cos\left(n^2_{\pi}-\mu_{\pi}\right)^2$

$= \sin(n^2\pi)\cos(n\pi) - \sin(n\pi)\cos(n^2\pi) - \sin\left(\frac{3\pi}{\mu}\right)\cdot\cos(n\mu-1)\pi)$

$= -\sin\left(\frac{3\pi}{\mu}\right)\left[\underbrace{\cos(n^2\pi)\cos(n\pi)}_{1} + \sin(n^2\pi)\sin(\mu\pi)\right]$

$\to \lim_{n\to+\infty} -n\,(\pm 1)\sin\left(\frac{3\pi}{\mu}\right) = \pm n\left(\frac{3\pi}{\mu} - \underbrace{\frac{9\pi^2}{6\mu^2}}_{0}\right)$

$= \pm 3\pi = -3\pi$

⑤ $\frac{x^2}{1-3x-x|x|} + 1 = 0$

- $x \geq 0 \quad \frac{x^2}{-1+3x+x^2} = -1 \to 0 = -1+3x+x^2$

  C.E.: $x \neq \frac{-3\pm\sqrt{9+4}}{2}$ ok $\qquad 3x = 1 \qquad x = 1/3$ 1 soluzione

- $x < 0 \quad \frac{x^2}{1-3x+x^2} = -1 \qquad x^2 = -1+3x-x^2$

  $2x^2 - 3x + 1 = 0$ sol. reali?

  $x \neq \frac{3\pm\sqrt{9-4}}{2}$

  $x = \frac{3\pm\sqrt{9-8}}{4} = 1,\ 1/2$ una $x<0$ non soluzione

  $x = 3/2 \pm \sqrt{\frac{9}{4}-\frac{8}{4}}$, $\sqrt{\frac{1}{2}}$

$\Rightarrow$ Ho una sola soluzione che è per $x \geq 0 \to x = 1/3$

⑥ fino a x^5 compreso

$\sin(\sin x) = \sin\left(x - \frac{x^3}{6} + \frac{x^5}{120}\right) = x - \frac{x^3}{6} + \frac{x^5}{120} + \frac{1}{6}\left(x - \frac{x^3}{6}\right)^3 + \frac{1}{120}x^5$

$= x - \frac{x^3}{6} + \frac{x^5}{60} + \frac{1}{12}x^5$

$-\arctg x = -x + \frac{x^3}{3} - \frac{x^5}{5}$

$\text{Num} = x^5\left(\frac{1}{60} - \frac{12}{60}\right) = x^5\left(\frac{11}{60}\right) = -\frac{11}{60}$

Den: $x^5 \cdot \frac{\pi}{2} \qquad \Rightarrow \lim = \frac{2}{\pi}\left(\frac{-1}{10}\right) = -\frac{1}{5\pi}$