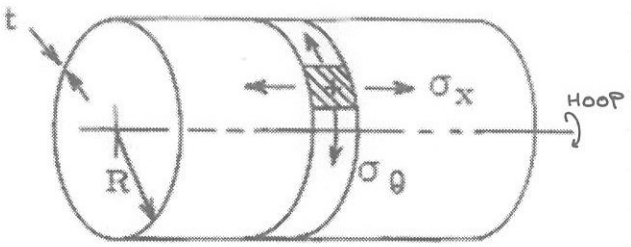
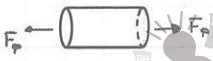
# THIN CIRCULAR PRESSURIZED TANKS

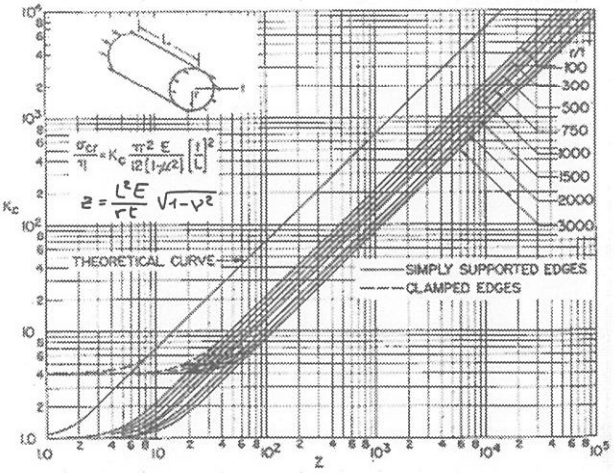



- HOOP STRESS → $\sigma_\theta = \frac{pR}{t}$
- LONGITUDINAL STRESS → $\sigma_x = \frac{pR}{2t}$
- RADIAL DISPLACEMENT → $w_R = \frac{pR^2}{Et}\left(1 - \frac{\nu}{2}\right)$

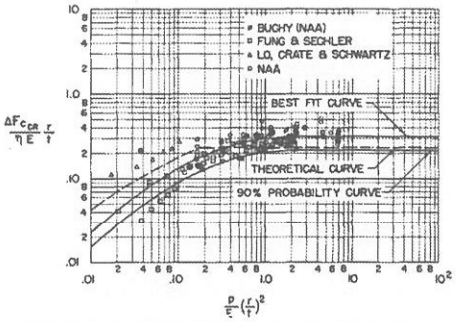
- PRESSURE RELIEF → $F_p = \pi R^2 p$




- BUCKLING (unpressurized) → $\frac{F_{cr}}{\eta} = K_c \frac{\pi^2 E}{12(1-\nu^2)}\left(\frac{t}{L}\right)^2$ ($K_c$ da grafici)

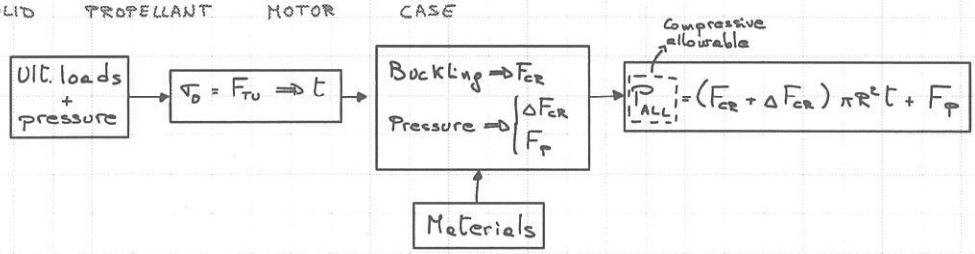



- INCREASE DUE TO $p \rightarrow \Delta F_{cr}$ (da grafici)




- EQUIVALENT AXIAL LOAD → $P_{eq} = P + \frac{2M}{R}$ [N] (Navier)

# SOLID PROPELLANT MOTOR CASE

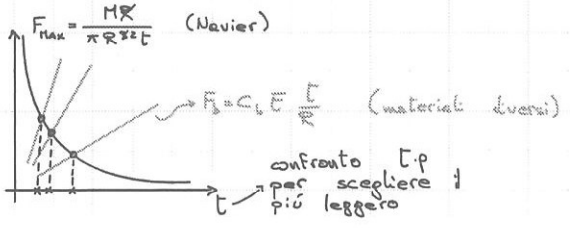


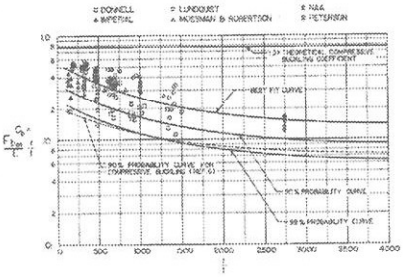

# LIQUID PROPELLANT MOTOR CASE

- PRESSURE FED → t maggiore, no pumps, $\Delta F_{CR}$ maggiore
- PUMP FED → t minore, pompe

- Procedimento analogo al precedente per i due casi, da confrontare poi i pesi

# INTERSTAGES / INTERTANKS



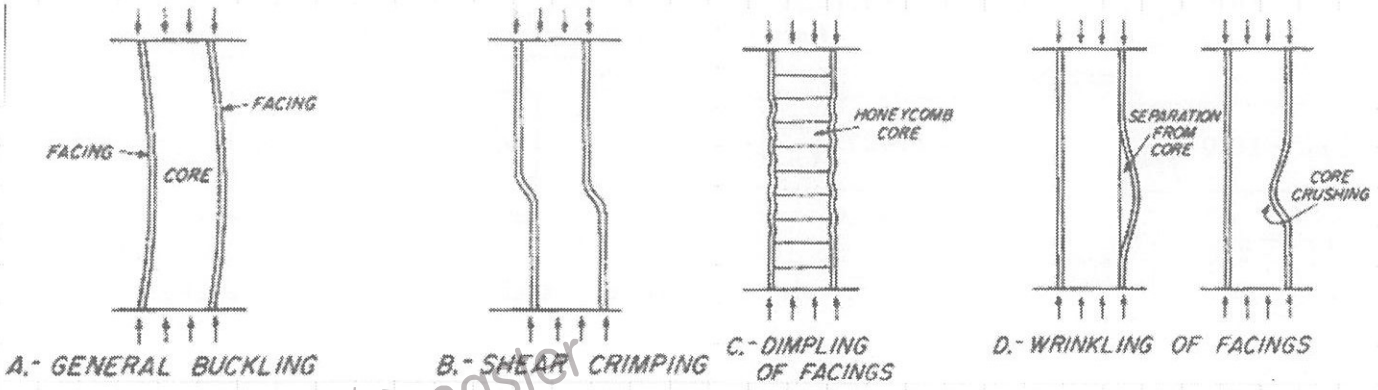




Si ricava $C_b$ per diversi t e si disegna $F_b(C_b)$ punto x punto


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LECTURE 5-7

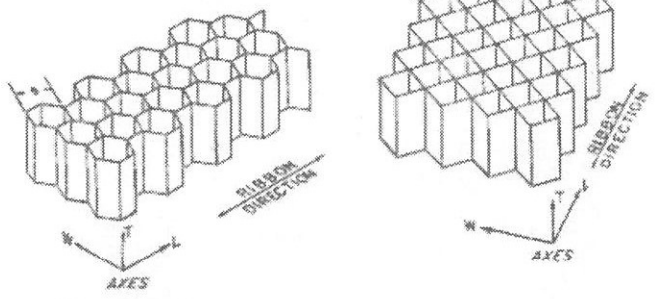



A.- GENERAL BUCKLING  B.- SHEAR CRIMPING  C.- DIMPLING OF FACINGS  D.- WRINKLING OF FACINGS

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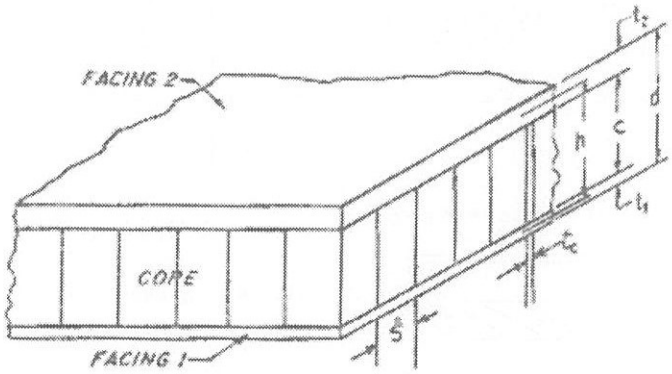
## Sandwich Construction and Design

HEXAGONAL  SQUARE




T = TRANSVERSE

HONEYCOMB CORES




SANDWICH CONSTRUCTION

* S = diametro del cerchio INSCRITTO

### Nomenclature used for cores and sandwich construction

PROPRIETÁ MECCANICHE (stiffness / width)

- TRANSVERSE BENDING (trascurando core)

$$D = \frac{E'_1 \cdot t_1 \cdot E'_2 \cdot t_2 \cdot h^2}{(E'_1 \cdot t_1 + E'_2 \cdot t_2) \cdot (1 - \nu^2)} \quad \text{(for unequal facings)}$$

$$D = \frac{E' \cdot t \cdot h^2}{2 \cdot (1 - \nu^2)} \quad \text{(for equal facings)}$$

*thin facings*

$$D = \frac{E'_1 \cdot t_1 \cdot E'_2 \cdot t_2 \cdot h^2}{(E'_1 \cdot t_1 + E'_2 \cdot t_2) \cdot (1 - \nu^2)} + \frac{(E'_1 \cdot t_1^3 + E'_2 \cdot t_2^3)}{12 \cdot (1 - \nu^2)} \quad \text{(for unequal facings)}$$

$$D = \frac{E' \cdot t \cdot h^2}{2 \cdot (1 - \nu^2)} + \frac{E' \cdot t^3}{6 \cdot (1 - \nu^2)} \quad \text{(for equal facings)}$$

*thick facings*

master COPY

- E' → effective modulus of elasticity
- ν → Poisson's ratio

$$E'_1 = \frac{1}{t_1} \cdot \sum_i^{N1} E_{1i} \cdot t_{1i} \quad (\text{N1} = \text{number of layers of facing 1})$$

$$E'_2 = \frac{1}{t_2} \cdot \sum_i^{N2} E_{2i} \cdot t_{2i} \quad (\text{N2} = \text{number of layers of facing 2})$$

$$E' = \frac{1}{t} \cdot \sum_i^{N} E_i \cdot t_i \quad (\text{N} = \text{number of layers of the equal facings})$$

$$t = \sum_i^{N} t_i$$

- TRANSVERSE SHEAR

*thin facings* → $h \approx c$

$$U = \frac{h^2}{c} \cdot G'_c \approx h \cdot G'_c$$

(a) Thin facings on a thick core

$$U \approx d \cdot G'_c$$

(b) Generic expression

G'c → core shear modulus

18 $lb/ft^3 \cong 288.3\ kg/m^3$

$$\rho'_c < 18.0\ \frac{lb}{ft^3} \Rightarrow F'_s = 1.307 \cdot \left(\frac{\rho'_c}{\rho_c}\right)^{1.34} \cdot \frac{F_{su}}{c^{0.44}} \qquad G'_c = 2.43 \cdot \left(\frac{\rho'_c}{\rho_c}\right)^{1.54} \cdot G_c$$

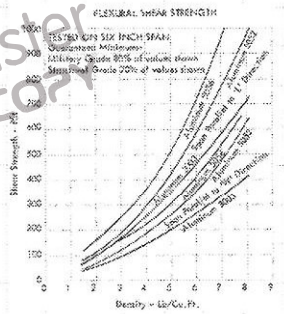
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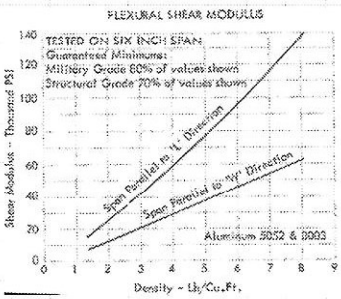
$$\rho'_c \geq 18.0\ \frac{lb}{ft^3} \quad F'_s = 0.427 \cdot \left(\frac{\rho'_c}{\rho_c}\right) \cdot \frac{F_{su}}{c^{0.44}} \qquad G'_c = 0.4 \cdot \left(\frac{\rho'_c}{\rho_c}\right) \cdot G_c$$

CORE DENSITY — CORE SHEAR STRENGTH — CORE SHEAR MODULUS

$$\begin{cases} \rho'_c = \dfrac{2 \cdot t_c}{s} \cdot \rho_c & \text{SQUARE CELL} \\ \rho'_c = \dfrac{3 \cdot t_c}{s} \cdot \rho_c & \text{HEXAGONAL CELL} \end{cases}$$

Formule empiriche, sostituibili da grafici

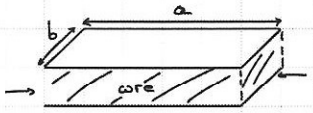



- CORE EFF. YOUNG MODULUS

$$\begin{cases} E'_c = 2.13\left(\dfrac{\rho'_c}{\rho_c}\right)^{1.415} \cdot E_c & \text{(empirico)} \\ E'_c = \dfrac{\rho'_c}{\rho_c} E_c & \text{(teorico)} \end{cases}$$

UNIAXIAL COMPRESSION




- BUCKLING GLOBAL EQUATION → $\dfrac{F_{ccr}}{\eta} = \dfrac{K_c}{J}\dfrac{U}{2t_F}$ (critical stress; buckling coefficient; plasticity)

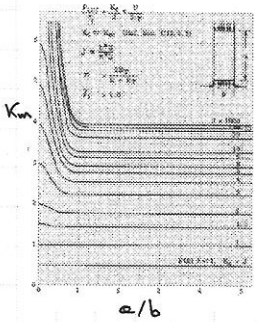
$F_i = F_{ccr}$

where $J = \dfrac{b^2 U}{\pi^2 D}$

$$\begin{cases} U = G'_c(c + 2t_F) \\ D = \dfrac{E_F t_F (c+t_F)^2}{2(1-\nu^2)} \end{cases}$$

- BUCKLING COEFFICIENT → $K_c = K_m + K_F$ (di solito $K_F$ trascurabile; $K_m$ da grafici)

S.S. PANELS — CLAMPED



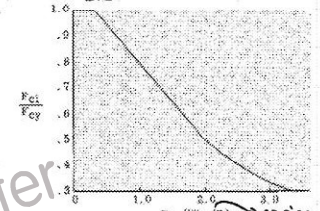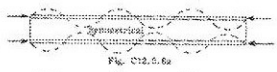




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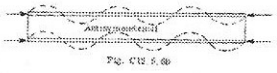



- LOCAL INSTABILITY
  - INTRACELL BUCKLING (thin faces)
  - WRINKLING (light $\rho'_c$)

- INTRACELL BUCKLING (= dimpling) (Failure of faces) → $\dfrac{F_{ci}}{\eta} = 0.75 \cdot E_F \cdot \left(\dfrac{t_F}{s}\right)^{\frac{3}{2}}$ where $\eta = \dfrac{2E_T}{E_F + E_T}$

- WRINKLING → $\dfrac{F_{cw}}{\eta_2} = 0.43 \cdot \sqrt[3]{E_F \cdot E'_c \cdot G'_c}$ (honeycomb, ribbon direction) where $\eta_2 = \sqrt[3]{\dfrac{3E_T + E_{sc}}{4E_F}}$ (Ramberg-Osgood)







- DIMPLING compression → $F_c = 0.75 \cdot E'_F \cdot \left(\dfrac{t_F}{s}\right)^{3/2}$ where $E'_F = \eta E_F$

$\Rightarrow s = 0.825\, t_F\, (F_c/E'_F)^{-2/3}$

N.B. Non è una failure mode (se non molto pronunciato) ma può portare a deform. plastica

## SHEAR BUCKLING

- BUCKLING GENERAL EQUATION → $\frac{F_s}{\eta} = \frac{K_s}{J}\frac{U}{2t_F}$ (critical stress; buckling coefficient (da grafici come $K_m$); plasticity) where $\eta = \frac{2E_T}{E+E_T}$

  $F_i = \sqrt{3}\,F_{cr}$ (Von Mises)

- DIMPLING shear → $F_c = 0.60 \cdot E'_F \cdot \left(\frac{t_F}{s}\right)^{3/2}$ where $E'_F = \eta E_F$

  ⇒ $s = 0.71\, t_F \left(F_c/E'_F\right)^{-2/3}$

- SHEAR CRIMPING → $F_c = \frac{U}{2t_F} = \frac{G'_c(c+2t_F)}{2t_F}$

## CURVED/CYLINDRICAL PANELS

Formule analoghe, si definiscono $K_x$ (buckling coeff.), riportati in slides 30 → 34 di "Lecture 6.1"

## DESIGN PROCEDURE

(i) $F_c \rightarrow t_F^{(min)}$

(ii) Buckling → $c^{(min)}$ (⇐ $D^{(min)}$ ⇐ general buckling eq.)

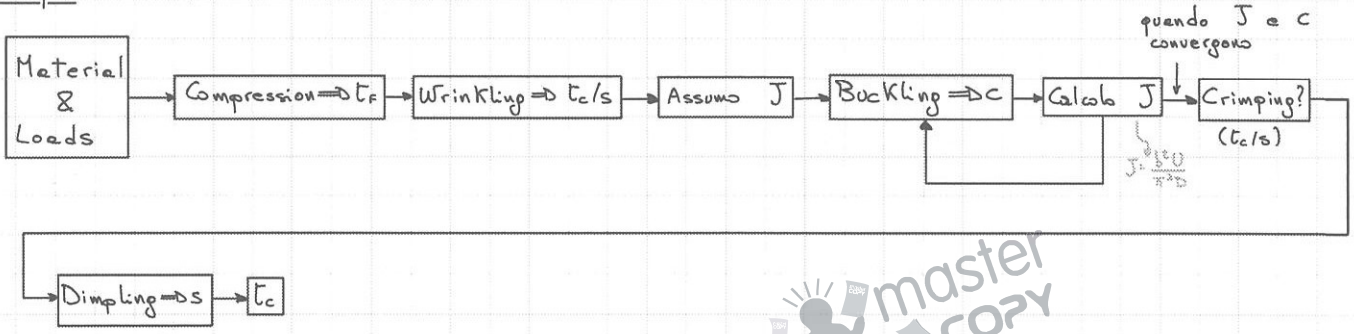
(iii) Wrinkling → $E'^{(min)}_c$ (⇒ $\frac{t_c}{s}$ min)

(iv) Dimpling → $s^{(max)}$ (if honeycomb)

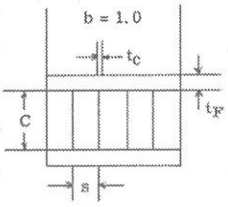
(v) Buckling (eulerian) → Entire cylinder bending stifness

} FLAT PANELS } CYLINDERS

## <u>example</u> (PANEL IN COMPRESSION)




## <u>example</u> (COLUMN ELEMENT)




CROSS SECTION → $A = b(2t_F + 2\frac{t_c}{s}c)$

Wrinkling + Eulerian inst. ⇒ $A(t_F)$ ← minimizzare