$$E(t) = \frac{\sigma(t)}{\varepsilon_0} = \frac{Q_0}{S\varepsilon_0} e^{-t/Rc}$$

$$\left| \varepsilon \frac{\partial \bar{E}}{\partial t} \right| = \left| \varepsilon \frac{Q_0}{S\varepsilon_0} \frac{1}{Rc} e^{-t/Rc} \right|$$

$$\downarrow \frac{1}{S} \frac{Q_0}{Rc} e^{-t/Rc}$$

$$I_{sf} = \int_S \bar{J} \cdot d\bar{S} = J \cdot S = \cancel{S} \cdot \frac{1}{\cancel{S}} \cdot \frac{Q_0}{Rc} e^{-t/Rc}$$

$$\boxed{I_{sp} = \frac{Q_0}{Rc} e^{-t/Rc}}$$

$$\boxed{I_c = \frac{Q_0}{Rc} e^{-t/Rc}}$$

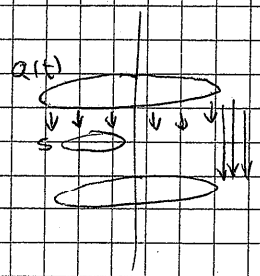
$$\boxed{\bar{\nabla} \times \bar{B} = \mu_0 \left(\bar{J} + \varepsilon \frac{\partial \bar{E}}{\partial t} \right)}$$

$$\boxed{\oint_\ell \bar{B} \cdot d\bar{\ell} = \mu_0 I^c + \mu_0 \varepsilon \int_S \frac{\partial \bar{E}}{\partial t} \cdot d\bar{S}}$$

Quindi ora posso usare il teorema di Ampere su tutte le superfici

Quindi nel caso del condensatore

$$\oint_\ell \bar{B} \cdot d\bar{\ell} = \mu_0 \varepsilon_0 \int_S \frac{\partial \bar{E}}{\partial t} d\bar{S}$$



$$2\pi R B_\phi(R) = \mu_0 \varepsilon \pi R^2 \frac{dE}{dt}$$

$$\boxed{B_\phi(R) = \frac{1}{2} \mu_0 \varepsilon_0 \frac{dE}{dt}}$$

$$\boxed{\bar{\nabla} \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}}$$

$$\boxed{\oint_\gamma \bar{E} \cdot d\bar{\ell} = -\frac{\partial \Phi(\bar{B})}{\partial t}}$$

$$\downarrow$$

$$2\pi R E_\phi(R) = -\pi R^2 \frac{dB}{dt}$$

$$\boxed{E_\phi(R) = -\frac{1}{2} R \frac{dB}{dt}}$$

$$\boxed{\begin{aligned} &\bar{\nabla} \cdot \bar{E} = \rho/\varepsilon_0 \qquad \bar{\nabla} \cdot \bar{B} = 0 \\ &\bar{\nabla} \times \bar{E} = -\frac{\partial \bar{B}}{\partial t} \qquad \bar{\nabla} \times \bar{B} = \mu_0 \left(\bar{J}_c + \varepsilon \frac{\partial \bar{E}}{\partial t} \right) \end{aligned}}$$

$$\frac{dQ}{dt} = \varepsilon S \frac{\partial E}{\partial t} = S \varepsilon \frac{\partial E}{\partial t}$$

$$\vec{J}_{sp} = \varepsilon \frac{\partial \vec{E}}{\partial t}$$

$$\vec{J}_{TOT} = \vec{J}_c + \vec{J}_{sp}$$

$\vec{J}_c$: J di conduzione; $\vec{J}_{sp}$: J di spostamento

Tutto ciò è compatibile con le equazioni di continuità?

$$\vec{\nabla} \cdot \vec{J} + \frac{\partial \rho}{\partial t} = 0$$

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \longrightarrow \rho = \varepsilon \vec{\nabla} \cdot \vec{E}$$

$$\vec{\nabla} \cdot \vec{J} + \varepsilon \frac{\partial}{\partial t} \vec{\nabla} \cdot \vec{E} = 0$$

$$\vec{\nabla} \cdot \vec{J} + \varepsilon \vec{\nabla} \left(\frac{\partial \vec{E}}{\partial t} \right) = 0$$

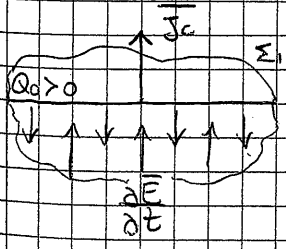
$$\vec{\nabla} \left(\vec{J}_c + \varepsilon \frac{\partial \vec{E}}{\partial t} \right) = 0$$

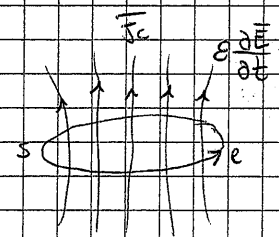
$$\boxed{\vec{J}_T = \vec{J}_0 + \varepsilon \frac{\partial \vec{E}}{\partial t}} \longrightarrow \boxed{\vec{\nabla} \cdot \vec{J}_T = 0}$$

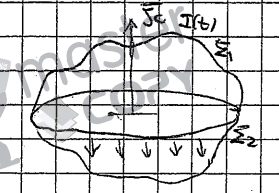
$$\boxed{\vec{\nabla} \times \vec{B} = \mu_0 \vec{J}_c + \mu_0 \varepsilon \frac{\partial \vec{E}}{\partial t}}$$ IV Maxwell

Così modificando anche il Teorema di Ampere:

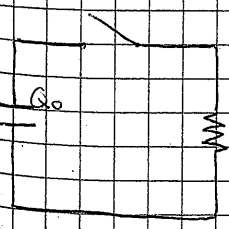
$$\oint_\ell \vec{B} \cdot d\vec{\ell} = \int_S \mu_0 \left(\vec{J}_c + \varepsilon \frac{\partial \vec{E}}{\partial t} \right) \cdot d\vec{S}$$







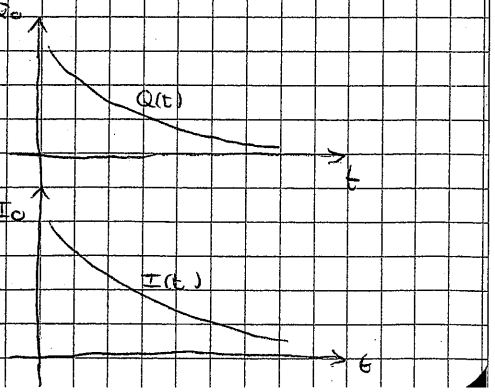
Riprendiamo il problema del condensatore



$$Q(t) = Q_0 e^{-t/Rc}$$

$$I(t) = \frac{dQ}{dt}$$

$$I(t) = \frac{Q_0}{Rc} e^{-t/Rc}$$



L'equazione del teorema di Gauss va bene anche per il caso non stazionario:

$$\nabla \cdot \vec{B} = 0$$

$$\oint \vec{B} \cdot d\vec{S} = 0$$

Anche questa equazione non ha problemi in condizioni non stabili:

$$\nabla \times \vec{E} = 0$$

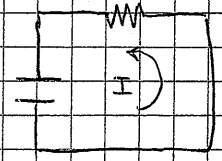
Questa già l'avevamo aggiustata

Il problema è nella seguente:

$$\nabla \times \vec{B} = \mu_0 \vec{J}$$

$$\oint_\gamma \vec{B} \cdot d\vec{\ell} = \mu_0 I^c$$

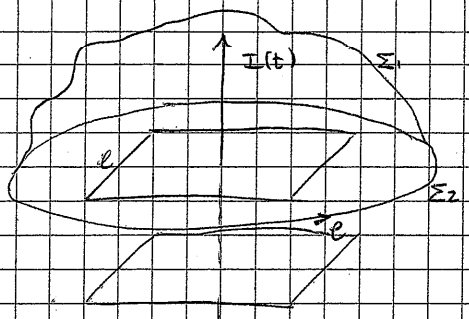
Guardiamo un circuito R-C per capire meglio:



Osserviamo la scarica del condensatore:

$$Q(t) = Q_0 e^{-t/Rc}$$

$$I(t) = \frac{Q_0}{Rc} e^{-t/Rc}$$



CONDENSATORE

Per Ampere:

$$\oint_\gamma \vec{B} \cdot d\vec{\ell} = \mu_0 I^c$$

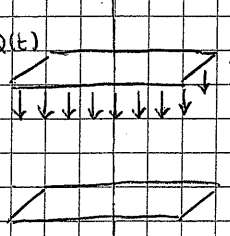
$$\int_{\Sigma_1} \vec{J} \cdot d\vec{S} = I(t)$$

$$\int_{\Sigma_2} \vec{J} \cdot d\vec{S} = 0$$

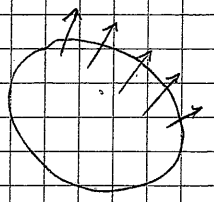
Quindi il teorema di Ampere non va bene per ogni superficie e non ho una definizione corretta di corrente.

In generale:

$$\int_S \vec{J} \cdot d\vec{S} = -\frac{dQ}{dt}$$



$$E(t) = \frac{\sigma(t)}{\varepsilon_0}$$



$$E(t) = \frac{\sigma(t)}{\varepsilon} \qquad \sigma = \frac{Q}{S}$$

$$E = \frac{Q(t)}{\varepsilon S}$$

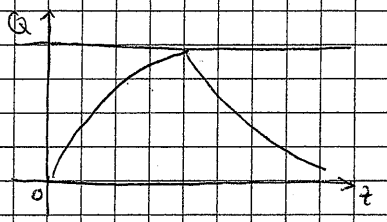
$$Q(t) = \varepsilon S E(t)$$

$$\mathcal{f} = \frac{Q}{C} + R\frac{dQ}{dt}$$

$$0 = \frac{Q}{C} + R\frac{dQ}{dt}$$

$$Q(0) = C\mathcal{f}$$

$$Q(t) = C\mathcal{f}e^{-t/Rc}$$



Analisi energetica del circuito Rc

$$\left(\mathcal{f} = \frac{Q}{C} + RI\right)I$$

$$W_G = W_C + W_R$$

$$W_G = \mathcal{f}I = \frac{\mathcal{f}^2}{R}e^{-t/Rc} \xrightarrow[t\to\infty]{} 0$$

$$W_R = \frac{Q}{C}I = \frac{\mathcal{f}^2}{R}\left(1 - e^{-t/Rc}\right)e^{-t/Rc} \xrightarrow[t\to\infty]{} 0$$

$$U_C = \frac{Q^2}{2C}$$

$$W_R = RI^2 = R\frac{\mathcal{f}^2}{R^2}\left(e^{-t/Rc}\right)^2 \xrightarrow[t\to\infty]{} 0$$

$$0,T \qquad \int_0^t W_R\,dt$$

Equazioni di Maxwell

$$\vec{\nabla}\cdot\vec{E} = \frac{\rho_c}{\varepsilon_0} \longrightarrow \oint_S \vec{E}\cdot d\vec{S} = \frac{Q_{INT}}{\varepsilon_0}$$

$$\vec{\nabla}\times\vec{E} = 0 \longrightarrow \vec{\nabla}\times\vec{E} = -\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\cdot\vec{B} = 0 \longrightarrow \oint_S \vec{B}\cdot d\vec{S} = 0$$

$$\vec{\nabla}\times\vec{B} = \mu_0\vec{J} \longrightarrow \oint_\gamma \vec{B}\cdot d\vec{e} = \mu_0 I^c$$

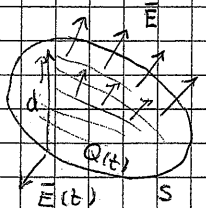
Caso non stazionario:

$\vec{\nabla}\cdot\vec{E} = \rho/\varepsilon_0$ Se le cose dipendono da t

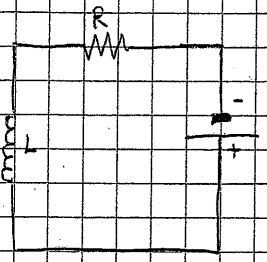
Cosa accade al teorema di Gauss quando $\vec{E}$ dipendenza da t:

$$\oint_S \vec{E}(t)\cdot d\vec{S} = \frac{Q_{INT}(t)}{\varepsilon_0}$$

$$\oint_S \vec{E}(t)\cdot d\vec{S} = \int_V \frac{\rho(x,t)}{\varepsilon}\,dV$$



Analisi energetica di un circuito RL



$$\left(f = RI + L\frac{dI}{dt}\right) \cdot I\,dA$$

$$I(t) = \frac{f}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

$W_G = W_R + W_L$ = potenza dissipata totale

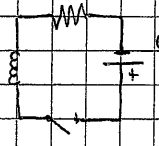
$$W_G = fI = \frac{f^2}{R}\left(1 - e^{-\frac{R}{L}t}\right)$$

$$W_R = RI^2 = R\left(\frac{f}{R}\left(1 - e^{-\frac{R}{L}t}\right)\right)^2$$

$$W_L = LI\frac{dI}{dt} = \frac{f^2}{R}\left(1 - e^{-\frac{R}{L}t}\right)e^{-R/L\,t} \longrightarrow 0$$

$$\frac{dI}{dt} = \frac{f}{L}e^{-\frac{R}{L}t}$$

$$f = RI + L\frac{dI}{dt}$$



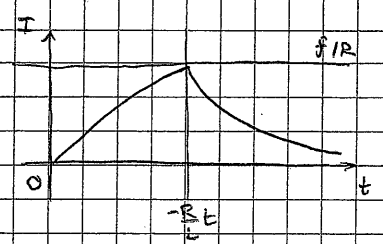
→ stacco generatore

$$I(0) = 0$$

$$I(t) = \frac{f}{R}\left(1 - e^{-R/L\,t}\right)$$

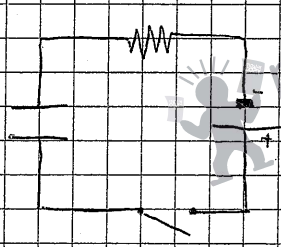
$$0 = RI + L\frac{dI}{dt} \quad ①$$

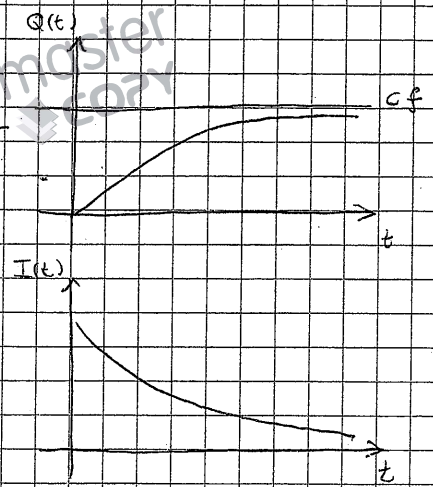
$$I(0) = f/R$$



Carica e scarica del condensatore

$$\begin{cases} f = \dfrac{Q}{C} + RI \\ Q(0) = 0 \end{cases}$$





$$f = \frac{Q}{C} + R\frac{dQ}{dt}$$

$$Q_0(t) = Ae^{-t/Rc}$$

$$Q_0 = cf_N$$

$$Q_0(0) = 0 \longrightarrow Q(t) = cf\left(1 - e^{-t/Rc}\right)$$

$$I(t) = \frac{dQ}{dt} = \frac{f}{R}e^{-t/Rc}$$