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GRADI	RADIANTI	SENO	COSENO	TANGENTE	COTANGENTE
$0°$	0	0	1	0	
$15°$	$\frac{\pi}{12}$	$\frac{\sqrt{6}-\sqrt{2}}{4}$	$\frac{\sqrt{6}+\sqrt{2}}{4}$	$2-\sqrt{3}$	$2+\sqrt{3}$
$30°$	$\frac{\pi}{6}$	$\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$
$45°$	$\frac{\pi}{4}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$	1	1
$60°$	$\frac{\pi}{3}$	$\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$
$90°$	$\frac{\pi}{2}$	1	0		0
$120°$	$\frac{2}{3}\pi$	$\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$-\sqrt{3}$	$-\frac{\sqrt{3}}{3}$
$135°$	$\frac{3}{4}\pi$	$\frac{\sqrt{2}}{2}$	$-\frac{\sqrt{2}}{2}$	-1	-1
$150°$	$\frac{5}{6}\pi$	$\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	$-\sqrt{3}$
$180°$	π	0	-1	0	
$210°$	$\frac{7}{6}\pi$	$-\frac{1}{2}$	$-\frac{\sqrt{3}}{2}$	$\frac{\sqrt{3}}{3}$	$\sqrt{3}$
$240°$	$\frac{4}{3}\pi$	$-\frac{\sqrt{3}}{2}$	$-\frac{1}{2}$	$\sqrt{3}$	$\frac{\sqrt{3}}{3}$
$270°$	$\frac{3}{2}\pi$	-1	0		0
$300°$	$\frac{5}{3}\pi$	$-\frac{\sqrt{3}}{2}$	$\frac{1}{2}$	$-\sqrt{3}$	$-\frac{\sqrt{3}}{3}$
$330°$	$\frac{11}{6}\pi$	$-\frac{1}{2}$	$\frac{\sqrt{3}}{2}$	$-\frac{\sqrt{3}}{3}$	$-\sqrt{3}$
$360°$	2π	0	1	0	

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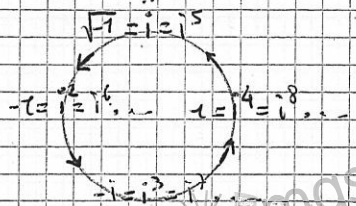
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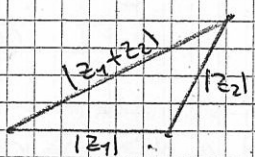
Numeri Complessi

- def. di unità immaginaria: $i = \sqrt{-1}$
- Insieme dei numeri complessi: $\mathbb{C}$
- prop. fondamentale: $i^2 = -1$



Numero complesso: $z = \underbrace{x}_{Re(z)} + i\underbrace{y}_{Im(z)}$

- Coniugio (coniugato): $\overline{z} = x - iy$
  - proprietà: ① $\overline{(\overline{z})} = z$ doppio coniugato
    ② $\overline{(z_1 + z_2)} = \overline{z}_1 + \overline{z}_2$ somma
    ③ $\overline{(z_1 \cdot z_2)} = \overline{z}_1 \cdot \overline{z}_2$ prodotto
- MODULO: $|z| = \sqrt{x^2 + y^2}$
  - proprietà: ① $|z| \in \mathbb{R} \geq 0 \rightarrow$ se $|z| = 0 \Rightarrow z = 0$
    ② $|z_1 + z_2| \leq |z_1| + |z_2|$ Disuguaglianza triangolare
    ③ $|z_1 \cdot z_2| = |z_1| \cdot |z_2|$ prodotto



- Regole: ① $|z|^2 = z \cdot \overline{z}$
  ② $z \cdot \left(\frac{\overline{z}}{|z|^2}\right) = 1$
  ③ IL RECIPROCO: $z^{-1} = \frac{\overline{z}}{|z|^2}$

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- forma trigonometrica: $z = \underbrace{|z|}_{\text{modulo } (\sqrt{x^2+y^2})}(\cos\vartheta + i\,\text{sen}\,\vartheta) \Rightarrow |z|e^{i\vartheta}$
  - argomento: $\vartheta = \tan^{-1}\frac{y}{x}$

- Operazioni dei numeri complessi.

ADDIZIONE e SOTTRAZIONE: $z_1 \pm z_2 = (x_1 + x_2) \pm i(y_1 + y_2)$

MOLTIPLICAZIONE: - forma algebrica: $z_1 \cdot z_2 = (x_1 x_2 - y_1 y_2) + i(x_1 y_2 + x_2 y_1)$
- forma trigonometrica: $z_1 \cdot z_2 = |z_1| \cdot |z_2|(\cos(\vartheta_1 + \vartheta_2) + i\,\text{sen}(\vartheta_1 + \vartheta_2))$

DIVISIONE: - forma algebrica: $\frac{z_1}{z_2} = \frac{x_1 x_2 + y_1 y_2}{x_2^2 + y_2^2} + i\frac{-x_1 y_2 + x_2 y_1}{x_2^2 + y_2^2}$
- forma trigonometrica: $\frac{z_1}{z_2} = \frac{|z_1|}{|z_2|}[\cos(\vartheta_1 - \vartheta_2) + i\,\text{sen}(\vartheta_1 - \vartheta_2)]$

POTENZA: $z^n = |z|^n[\cos(n\vartheta) + i\,\text{sen}(n\vartheta)]$

Formula di De Moivre: $\boxed{\cos\vartheta + i\,\text{sen}\,\vartheta = \cos(n\vartheta) + i\,\text{sen}(n\vartheta)}$

Funzione esponenziale

$\exp: \mathbb{C} \to \mathbb{C} \qquad z = x+iy \longmapsto e^z = e^{x+iy} = e^x \cdot e^{iy} = e^x(\cos(y) + i\,\text{sen}(y))$

$z \longmapsto e^z$

es. $z = 5 + i\frac{\pi}{3} \longmapsto e^z = e^5\left(\cos\left(\frac{\pi}{3}\right) + i\,\text{sen}\left(\frac{\pi}{3}\right)\right) = e^5\left(\frac{1}{2}\right) + i e^5 \frac{\sqrt{3}}{2} = \frac{e^5}{2} + i e^5 \frac{\sqrt{3}}{2}$

Esercizi

n. 1.3 pag 4

$z = 2-4i$

$w = 3+5i$

① $z+w = 2-4+3+5i = 5+i$

② $z-w = 2-4i-3-5i = -1-9i$

③ $|z+w| = |5+i| = \sqrt{25+1} = \sqrt{26}$

④ $z \cdot w = (2-4i)(3+5i) = (6-(-20)) + i(10-12) = 26-2i$

⑤ $\bar{z} \cdot w$ con $\bar{z} = 2+4i$

$\Rightarrow (2+4i)(3+5i) = (6-20) + i(10+12) = -16+22i$

⑥ $\frac{w}{z} = \frac{(2\cdot 3)+(-4)(5)}{4+16} + i\frac{-3(-4)+2(5)}{4+16} = -\frac{4}{5} + i\frac{11}{10}$

n. 1.6 a forma cartesiana

1) $z_1 = e^{i\frac{7}{2}\pi} = 1\left(\cos\left(\frac{7}{2}\pi\right) + i\,\text{sen}\left(\frac{7}{2}\pi\right)\right) = \emptyset - i$

2) $z_2 = e^{i\frac{2}{3}\pi} = 1\left(\cos\left(\frac{2}{3}\pi\right) + i\,\text{sen}\left(\frac{2}{3}\pi\right)\right) = -\frac{1}{2} + i\frac{\sqrt{3}}{2}$

3) $z_3 = e^{i\frac{\pi}{4}} = 1(\cos(\pi/4) + i\,\text{sen}(\pi/4)) = \frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}$

4) $z_4 = 5 \cdot e^{i\frac{3}{2}\pi} = 5(\cos(3/2\,\pi) + i\,\text{sen}(3/2\,\pi)) = \emptyset - 5i$

5) $z_5 = 4 \cdot e^{i\frac{2}{3}\pi} = 4(\cos(2/3\,\pi) + i\,\text{sen}(2/3\,\pi)) = 4\left(-\frac{1}{2} + i\frac{\sqrt{3}}{2}\right) = -2 + i2\sqrt{3}$

6) $z_6 = \sqrt{2} \cdot e^{i\frac{\pi}{4}} = \sqrt{2}(\cos(\pi/4) + i\,\text{sen}(\pi/4)) = \sqrt{2}\left(\frac{\sqrt{2}}{2} + i\frac{\sqrt{2}}{2}\right) = 1+i$

7) $z_7 = e^{5+i\frac{7}{2}\pi} = e^5(\cos(7/2\,\pi) + i\,\text{sen}(7/2\,\pi)) = e^5(\emptyset - i) = -ie^5$

8) $z_8 = e^{2+i\frac{\pi}{6}} = e^2\left(\cos\left(\frac{\pi}{6}\right) + i\,\text{sen}\left(\frac{\pi}{6}\right)\right) = e^2\left(\frac{\sqrt{3}}{2} + i\frac{1}{2}\right) = e^2\frac{\sqrt{3}}{2} + ie^2\frac{1}{2}$

9) $z_9 = e^{-1-i\frac{\pi}{2}} = \frac{1}{e}\left(\cos\left(\frac{\pi}{2}\right) - i\,\text{sen}\left(\frac{\pi}{2}\right)\right) = -\frac{i}{e} + \emptyset$

n. 1.5 a forma trigonometrica. $(|z|e^{i\vartheta})$

① $z_1 = -1$ $|z| = \sqrt{1} = 1$ $\cos\vartheta = \frac{x}{\sqrt{x^2+y^2}}$; $\operatorname{sen}\vartheta = \frac{y}{\sqrt{x^2+y^2}}$:

$\cos\vartheta = -1$ $\operatorname{sen}\vartheta = 0 \rightarrow \vartheta = \pi$

$\Rightarrow z_1 = e^{i\pi}$

② $z_2 = i\sqrt{3}$ $|z| = \sqrt{3}$ $\cos\vartheta = 0$ $\operatorname{sen}\vartheta = 1$ $\vartheta = \frac{\pi}{2} \Rightarrow z_2 = \sqrt{3}e^{i\pi/2}$

③ $z_3 = 18i$ $|z| = 18$ $\cos\vartheta = 0$ $\operatorname{sen}\vartheta = 1$ $\vartheta = \frac{\pi}{2} \Rightarrow z_3 = 18e$

④ $z_4 = 4+i$ $|z| = \sqrt{2}$ $\cos\vartheta = \frac{\sqrt{2}}{2}$ $\operatorname{sen}\vartheta = \frac{\sqrt{2}}{2}$ $\vartheta = \frac{\pi}{4} \Rightarrow z_4 = \sqrt{2}e^{i\pi/4}$

⑤ $z_5 = 2-2i\sqrt{3}$ $|z| = 4$ $\cos\vartheta = \frac{1}{2}$ $\operatorname{sen}\vartheta = -\frac{\sqrt{3}}{2}$ $\vartheta = \frac{5}{3}\pi \Rightarrow z_5 = 4e^{i5/3\pi}$

⑥ $z_6 = 6+i6\sqrt{3}$ $|z| = 12$ $\cos\vartheta = \frac{1}{2}$ $\operatorname{sen}\vartheta = \frac{\sqrt{3}}{2}$ $\vartheta = \frac{\pi}{3} \Rightarrow z_6 = 12e^{i\pi/3}$

⑦ $z_7 = \sqrt{3}+3i$ $|z| = 2\sqrt{3}$ $\cos\vartheta = \frac{1}{2}$ $\operatorname{sen}\vartheta = \frac{\sqrt{3}}{2}$ $\vartheta = \frac{\pi}{3} \Rightarrow z_7 = \sqrt{12}e^{i\pi/3}$

⑧ $z_8 = 2-2i$ $|z| = 2\sqrt{2}$ $\cos\vartheta = \frac{\sqrt{2}}{2}$ $\operatorname{sen}\vartheta = -\frac{\sqrt{2}}{2}$ $\vartheta = \frac{7}{4}\pi \Rightarrow z_8 = 2\sqrt{2}e^{i\frac{7}{4}\pi}$

⑨ $z_9 = -9$ $|z| = 9$ $\cos\vartheta = -1$ $\operatorname{sen}\vartheta = 0$ $\vartheta = \pi \Rightarrow z_9 = 9e^{i\pi}$

n. 1.6

$z_0 = 1+i$

ⓐ $z_0^2 = (1+i)^2 = 1+i^2+2i = 2i$

ⓑ $z_0^4 = (1+i)^4 = (2i)^2 = 4i^2 = -4$

ⓒ $z_0^8 = (1+i)^8 = (-4)^2 = 16$

n. 1.7 equazioni

$z_0 = 3+4i$

① $z+z_0 = 0 \rightarrow z = -z_0 = -3-4i$

② $z+2z_0 = 1 \rightarrow z = 1-2z_0 = 1-6-8i = -5-8i$

③ $3z-z_0 = 2+i \rightarrow 3z = 2+i+z_0 \rightarrow z = \frac{2+i+3+4i}{3} = \frac{5}{3}+\frac{5}{3}i = \frac{5}{3}(1+i)$

④ $z\cdot z_0 = 3 \rightarrow z = \frac{3}{z_0} = \frac{3}{3+4i} = \frac{3(3)+0(4)}{9+16} + i\frac{-3(4)+3(0)}{9+16} = \frac{9}{25} - i\frac{12}{25}$

⑤ $z-z_0 = 6+6i \rightarrow z = \frac{6+6i}{3+4i} = \frac{18+24}{9+16} + i\frac{-24+18}{9+16} = \frac{42}{25} - i\frac{6}{25}$

⑥ $\frac{z_0}{z} = 2+i \rightarrow z = \frac{3+4i}{2+i} = \frac{6+4}{4+1} + i\frac{-3+8}{5} = \frac{10}{5} + i = 2+i$