K	M	G	m	μ	n	p
$\times 10^3$	$\times 10^6$	$\times 10^9$	$\times 10^{-3}$	$\times 10^{-6}$	$\times 10^{-9}$	$\times 10^{-12}$
chilo	mega	giga	milli	micro	nano	pico

$l = 1{,}86\ m \quad \Delta t = 63$ $\qquad v = \frac{\Delta s}{\Delta t} = \frac{1{,}86\ m}{63\ s} = 0.0295\ m/s$ le cifre significative sono solo 3 vedi 1,86

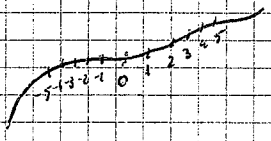
Δs

CINEMATICA → studio del movimento

↳ movimento di un punto geometrico proprio adimensionale privo di massa

Il moto può essere unidimensionale pur non essendo unidirezionale, esempio del treno Pisa-Lucca che ha le distanze espresse rispetto a Pisa indicata nel percorso Pisa, ovvero l'origine del sistema di riferimento, deve essere espresse, insieme al verso. In questo caso Pisa → Lucca +, Lucca → Pisa −

Traiettoria di un punto = insieme delle posizioni

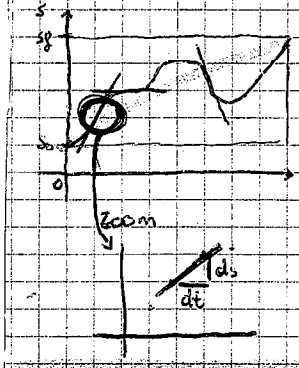


$s(t)$ $\quad \Delta s = s - s_0$ $\quad v = \frac{\Delta s}{\Delta t}$ questa è quella media

$\Delta t = t - t_0$

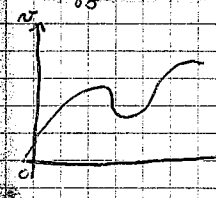
$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$

Legge oraria del moto



$v = \frac{ds}{dt}$ $\quad a_{medio} = \frac{\Delta v}{\Delta t}$ $\quad a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$

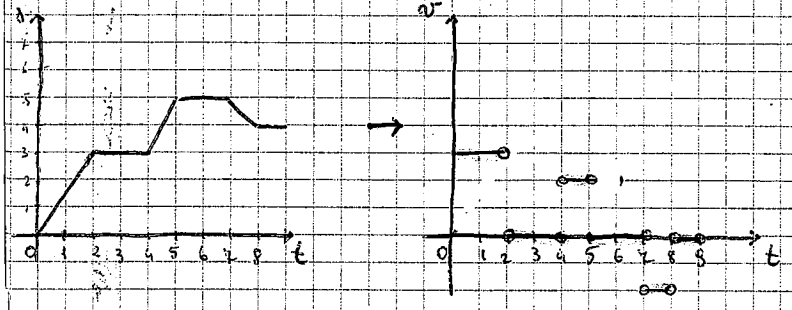
Legge oraria della velocità



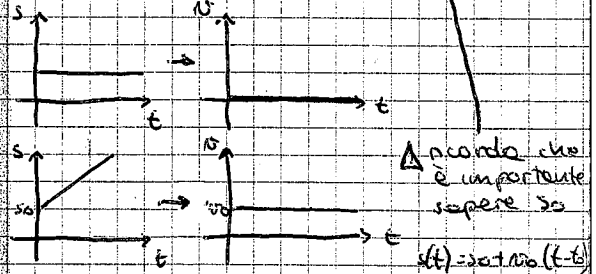
Δ non è il grafico del treno

Perché sono eq. differenziali del 2° ordine

Esercizio



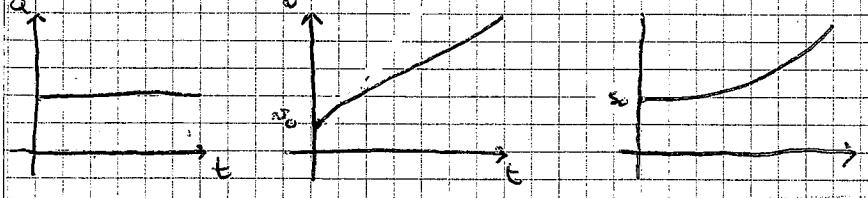
Esempi



Δ ricorda che è importante sapere s_0

$s(t) = s_0 + v_0 (t - t_0)$

esempi

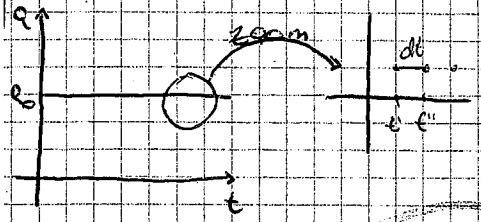


$v(t) = v_0 + a_0 (t - t_0)$ $\quad s(t) = s_0 + v_0 (t - t_0) + \frac{1}{2} a_0 (t - t_0)^2$

Moto uniformemente accelerato



MUA, percorso inverso



$$a = \frac{dv}{dt} \Rightarrow dv = a\,dt \rightarrow dv = v'' - v' = a\,dt$$

$$\Delta v = v'' - v_0 = \sum a\,dt = \int_0^{t''} a\,dt$$

$$v(t) = v_0 + \int_{t_0}^{t} a\,dt$$

$$v(t) = v_0 + a_0 (t - t_0) \quad ; \quad \int a_0\,dt = a_0 \int_{t_0}^{t} dt$$

$$\int_{t_0}^{t} dt = t - t_0$$

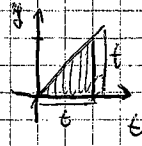
$$s(t) = s_0 + \int_{t_0}^{t} v\,dt \rightarrow s(t) = s_0 + \int_{t_0}^{t} \left[v_0 + a_0 (t - t_0) \right] dt$$

Radaltò classico

$$\int_{t_0}^{t} \left[v_0 + a_0 (t - t_0) \right] dt \quad t_0 = 0 \rightarrow \int_{t_0}^{t} \left[v_0 + a_0 t \right] dt = \int_{t_0}^{t} v_0\,dt + \int_{t_0}^{t} a_0 t\,dt =$$

$$v_0 \int_0^t dt + \int_0^t a_0 t\,dt = v_0 t + a_0 \int_0^t t\,dt = v_0 t + a_0 \frac{t^2}{2}$$

metodo alternativo per l'integrale di $\int t\,dt$



$y(t) = t$

Ripasso

$$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt} \qquad a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt} = \frac{d^2 s}{dt^2}$$

Moto uniformemente accelerato

$$s(t) = s_0 + v_0 (t - t_0) + \frac{1}{2} a (t - t_0)^2$$

Un altro esempio con i treni

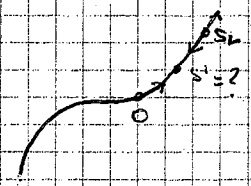
TRENO 1:

$t_0 = \emptyset$

viaggia a $v_{0x} = v_1$

TRENO 2:

a_2 costante verso Pisa parte fermo $\rightarrow v_{0_2} = 0$

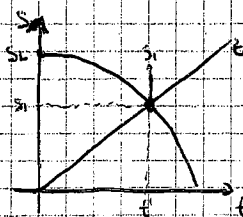


TRENO 1

$s_1(t) = s_{01} + v_1 t$

TRENO 2

$s_2(t) = s_2 + v_{02} t + \frac{1}{2} a_2 t^2$



$\exists t' \mid s_1(t = t') = s_2(t = t')$

$$\begin{cases} s' = v_1 t' \\ s' = s_2 + \frac{1}{2} a_2 t'^2 \end{cases} \qquad v_1 t' = s_2 + \frac{1}{2} a_2 t'^2$$

$$\frac{1}{2} a t'^2 - v_1 t' + s_2 = 0$$

$\frac{a}{2}t^2 - v_1 t + s_L = 0$

$t' = \frac{v_1 + \sqrt{v_1^2 - 2a_2 s_L}}{2a_2}$ facciamo la prova con le dimensioni

$\frac{[l]}{[t]} = \frac{[l] + \sqrt{\frac{[l]^2}{[t]^2} - \frac{[l]}{[t]^2}\cdot[l]}}{2\cdot\frac{[l]}{[t]^2}}$

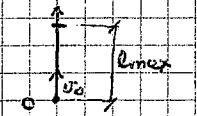
$= \frac{[l]}{[t]} = \frac{[l]}{[t]} \Big/ \frac{[l]}{[t]^2} = \frac{[l]}{[l]}\frac{[t]^2}{[t]} = [t]$ ✓

$s' = v_1 t'$

$s' = v_1 \cdot \frac{v_1 + \sqrt{v_1^2 - 2a_2 s_L}}{2a_2} = \frac{v_1^2 + \sqrt{v_1^2 - 2a_2 s_L}}{2a_2}$

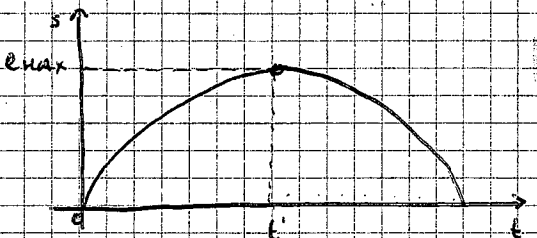
Un altro caso:

Supponiamo di lanciare un oggetto verso l'alto, quale sarebbe la sua altezza massima?



$y(t) = v_0 t + \frac{a}{2}t^2$ con $a = -g$

$t_0 = 0$



Cerco un istante $t' \mid v(t = t') = 0$

quindi $l_{MAX} = y(t = t')$

Devo scrivere anche la legge oraria delle velocità

$v(t) = v_0 + at$

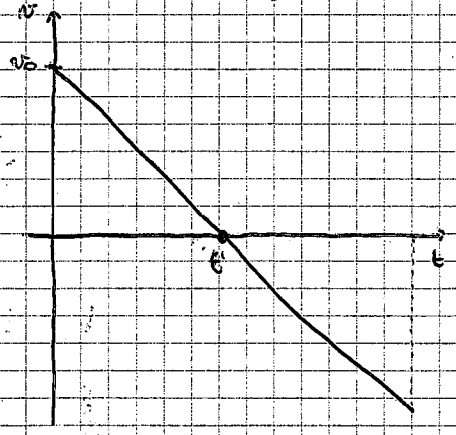
$0 = v_0 + at' \rightarrow t' = -\frac{v_0}{a}$

$y(t') = v_0\left(-\frac{v_0}{a}\right) + \frac{a}{2}\left(-\frac{v_0}{a}\right)^2 = -\frac{v_0^2}{a} + \frac{a}{2}\left(\frac{v_0^2}{a^2}\right) = -\frac{v_0^2}{a} + \frac{v_0^2}{2a} = \frac{-2v_0^2 + v_0^2}{2a} = -\frac{v_0^2}{2a}$

$= -\frac{v_0^2}{-2g} = +\frac{v_0^2}{2g}$

Parabola: vuol dire lanciare lontano, dal greco

Costruiamo il grafico della velocità



Domanda

↳ Velocità nell'istante in cui passo per la posizione iniziale

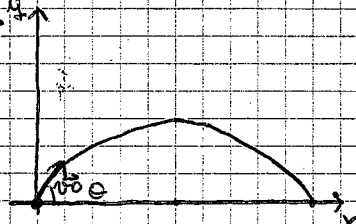
$v'' = ?$ è esattamente $-v_0$ [per il principio di cons. dell'E]

$t'' \mid y(t = t'') = 0$

$0 = v_0 t'' + \frac{1}{2}at'' = t''\left(v_0 + \frac{1}{2}at''\right) \rightarrow t'' = -\frac{2v_0}{a}$ (0 non acc.)

$v'' = v(t = t'') = v_0 - a\frac{2v_0}{a} = -v_0$ ✓

Moto di un proiettile



$x(t) = x_0 + v_x t - \frac{a}{2}t^2$

$y(t) = y_0 + v_y t + \frac{a_y}{2}t^2$

$t_0 = 0 \rightarrow x_0 = 0$, $y_0 = 0$, $a_x = 0$, $a_y = -g$

$\begin{cases} x(t) = v_{0x} t \\ y(t) = v_{0y} t + \frac{a_y}{2}t^2 \end{cases} \rightarrow t = \frac{x}{v_{0x}}$

$y = v_{0y}\frac{x}{v_{0x}} + \frac{a_y}{2}\left(\frac{x}{v_{0x}}\right)^2$

$= \frac{v_{0y}}{v_{0x}}x + \frac{a_y}{2v_{0x}^2}x^2$ ← parabola

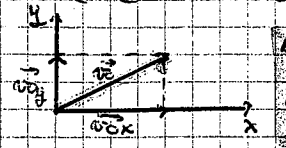


VETTORE: Grandezza fisica il cui valore dipende dalla direzione ed ha anche un verso

$|\vec{w}| = \dots$ m/s = lunghezza anche la posizione e l'accelerazione

$\vec{v_0} = (v_{0x}, v_{0y}, v_{0z})$ $\quad v_{0x} = |\vec{v_0}| \cos\theta$

$|\vec{v_0}| = \sqrt{v_{0x}^2 + v_{0y}^2 + v_{0z}^2}$ $\quad v_{0y} = |\vec{v_0}| \operatorname{sen}\theta$

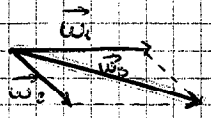


Δ graficamente prendiamo v_{0x} componente non $\vec{v_{0x}}$

OPERAZIONI CON I VETTORI

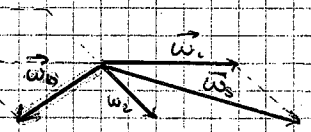
$\vec{w_1} = (w_{1x}, w_{1y}, w_{1z})$

$\vec{w_2} = (w_{2x}, w_{2y}, w_{2z})$ supponiamo che $z = 0$



$\vec{w_1} + \vec{w_2} = \vec{w}_{SOMMA} = (w_{1x} + w_{2x}; w_{1y} + w_{2y}; w_{1z} + w_{2z})$

$\vec{w_1} - \vec{w_2} = \vec{w_1} + (-\vec{w_2}) = \vec{w}_{DIFFERENZA}$



PUNTO DI APPLICAZIONE = "da dove partono le frecce"
↓ tipo la posizione — se un vettore ha il PdA non posso applicare la traslazione

LEGGI ORARIE DEL MOTO

$\begin{cases} x(t) = v_{0x} t \\ y(t) = v_{0y} t + \frac{1}{2} a t^2 \end{cases}$ $\quad t_0 = 0$

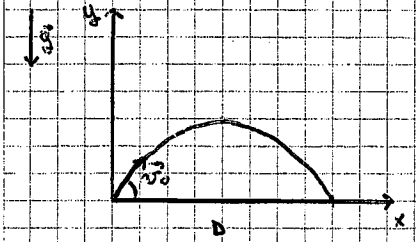
$a = -|\vec{g}|$

$s(t) = s_0 + v_0 t + \frac{1}{2} a t^2$

$\vec{r}(t) = \vec{r_0} + \vec{v_0} t + \frac{\vec{a}}{2} t^2$

$v = \lim_{\Delta t \to 0} \frac{\Delta s}{\Delta t} = \frac{ds}{dt}$; $\Delta \vec{r} = \vec{r} - \vec{r_0}$ $\quad \vec{v} = \frac{d\vec{r}}{dt}$

$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$; $\quad \vec{a} = \frac{d\vec{v}}{dt}$



$t' \mid y(t = t') = 0$

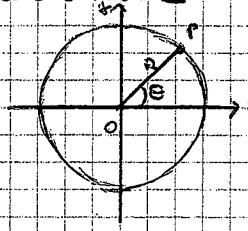
$D = x(t = t')$

$0 = v_{0y} t' - \frac{|\vec{g}|}{2} t'^2 \qquad t' < \frac{0}{\frac{2 v_{0y}}{|\vec{g}|}}$

$D = \frac{2 v_{0y} v_{0x}}{|\vec{g}|} = \frac{2 |\vec{v_0}|^2 \operatorname{sen}\theta \cos\theta}{|\vec{g}|} = \frac{|\vec{v_0}|^2 \operatorname{sen}(2\theta)}{|\vec{g}|}$

Δ Se io ho un cambio di velocità ho un'accelerazione
↓ posso cambiare direzione o modulo o entrambi

l'a è diretta ⊥ al moto



[coordinate polari] [nello spazio un punto può essere localizzato da un raggio e due angoli, vedi terra] [è anche con l'asse cilindrico]

$\hat{P}: 2\pi r = \theta : 2\pi$ $\quad \theta(t) = \theta_0 + \omega_0 t + \frac{1}{2} a t^2 \rightarrow R\theta(t) = R\theta_0 + R\omega_0 t + \frac{1}{2} a t^2 R$

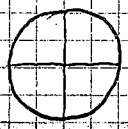
$\hat{P} = \frac{2\pi r \cdot \theta}{2\pi} = R \cdot \theta_{rad}$ $\quad R \cdot \omega : v = \frac{ds}{dt} = \frac{dR\theta}{dt} = R\frac{d\theta}{dt}$ $\quad \omega = \frac{d\theta}{dt}$ $\quad a = \frac{dv}{dt}$

master COPY

MOTO CIRCOLARE UNIFORME → ω cost

$t_0 = 0 \quad \theta(t) = \theta_0 + \omega_0 t$

$T \mid \theta(T) = 2\pi = \omega_0 \cdot T \rightarrow T = \frac{2\pi}{\omega_0}$

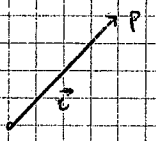


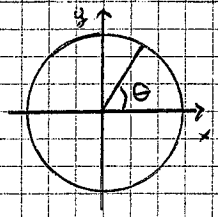
Rizzi

$\vec{r}$

$\vec{v} = \frac{d\vec{r}}{dt}$

$\vec{a} = \frac{d\vec{v}}{dt}$

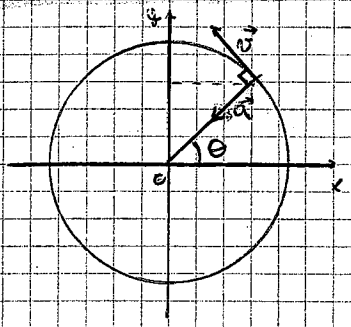




$\theta(t)$

$\frac{d\theta(t)}{dt} = \omega(t)$ velocità angolare $= \dot{\theta}(t)$

$\frac{d\omega}{dt} = \alpha(t)$ accelerazione angolare $= \dot{\omega} = \ddot{\theta}$



$\vec{r}(t) : (R\cos\theta ; R\operatorname{sen}\theta)$ derivare un vettore vuol dire:

$\vec{v} = \frac{d\vec{r}}{dt}$

$\vec{a} - \vec{b} : (a_x - b_x, a_y - b_y)$

cioè fare la derivata componente per componente

Quindi la derivata di

$(R\cos(\theta(t)) ; R\operatorname{sen}(\theta(t))) = \left(-R\operatorname{sen}(\theta(t))\frac{d\theta}{dt}, R\cos\theta(t)\frac{d\theta}{dt}\right) =$

$= R\omega(t)\left(-\operatorname{sen}(\theta(t)), \cos(\theta(t))\right)$

raggio R

$|\vec{v}| = R\omega$

PRODOTTO SCALARE

$\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}|\cos\theta$ se $\vec{a} \perp \vec{b} \Rightarrow \vec{a} \cdot \vec{b} = 0$

$\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y$



MOTO CIRCOLARE UNIFORME

$\omega = \frac{d\theta}{dt} = \text{COSTANTE} \qquad \vec{a} = -R\omega\left(\cos(\theta(t))\frac{d\theta}{dt}, \operatorname{sen}(\theta(t))\frac{d\theta}{dt}\right)$

$= -\omega^2 R(\cos\theta, \operatorname{sen}\theta)$

$= -\omega^2 \cdot \vec{r}$

VERSORE = vettore con lunghezza 1 $|\hat{v}| = 1$ adimensionale

Come creare un versore → $\vec{a}$, $\hat{a} = \frac{\vec{a}}{|\vec{a}|} \rightarrow \vec{a} = |\vec{a}|\,\hat{a}$

Nel piano cartesiano solitamente ci sono

$\hat{i}(1,0,0)$ ↓ x $\quad \hat{j}(0,1,0)$ ↓ y $\quad \hat{k}(0,0,1)$ ↓ z sono tutti ortogonali tra loro

Possiamo definire due versori $\hat{r}$, $\hat{\theta}$ il primo radiale, il secondo tangenziale in senso antiorario

