9 | 06 | 2020

Ejercicio 2:

$$\begin{pmatrix} p & & 1 \\ \alpha & \ddots & \vdots \\ \alpha & 0 & p \end{pmatrix}$$

1. $|p| > 1$

$|p| > |\alpha| + 1$

$|p| > |\alpha|$

$|\alpha| < |p| - 1$

$-p+1 < \alpha < p-1$

2. COND. NEC. & SUFF. $\rho(P) < 1$

GAUSS-RADEL: $A = D - L - U$

$M = D - L \quad N = U$

$P = M^{-1} N \qquad M = \begin{pmatrix} p & & \\ +\alpha & \ddots & \\ -\alpha & & p \end{pmatrix} \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix} = \begin{pmatrix} 1 \\ \vdots \\ 0 \end{pmatrix}$

$x_1 = 1/p$

$+\alpha x_1 + p x_2 = 0 \quad x_2 = -\alpha/p^2$

$+\alpha x_2 + p x_3 = 0 \quad x_3 = -\alpha/p^2$

$\rightarrow M^{-1} = \begin{pmatrix} -\alpha/p^2 & 1/p & 0 \\ \vdots & \ddots & 1/p \\ -\alpha/p^2 & 0 & \end{pmatrix}$

$p x_1 = 0$

$+\alpha x_2 + p x_2 = 1 \quad \rightarrow x_1 = 1/p$

$+\alpha x_1 + p x_3 = 0 \quad x_3 = 0$

$+\alpha x_1 + p x_4 = 0 \quad x_4 = 0$

$p x_1 = 0$

$-\alpha x_1 + p x_2 = 0 \quad x_2 = 0$

$-\alpha x_1 + p x_3 = 1 \quad x_3 = \frac{1}{p}$

$p x_2 = 0$

$-\alpha x_1 + p x_n = 1$

$$M^{-1} N \begin{pmatrix} -\alpha/p^2 & 1/p & 0 \\ \vdots & \ddots & 1/p \\ -\alpha/p^2 & 0 & \end{pmatrix} \begin{pmatrix} -1 \\ \vdots \\ 0 \end{pmatrix} = \begin{pmatrix} & +\alpha/p^2 & \\ 0 & +\alpha/p^2 - 1 & \\ & \vdots & \\ & +\frac{\alpha}{p} - 1 & \end{pmatrix}$$

$$\begin{pmatrix} & & +\alpha/p^2 \\ & & +\alpha/p^1-1 \\ 0 & & \vdots \\ & & +\alpha/p^{i}-1 \\ & & +\frac{\alpha}{p^n} \end{pmatrix}$$

$$\det(P-\lambda I) = \begin{pmatrix} -\lambda & & +\alpha/p^2 \\ & \ddots & +\alpha/p^1-1 \\ & & -\lambda+\alpha/p^{i}-1 \\ & & -\lambda+\frac{\alpha}{p^n} \end{pmatrix}$$

$$(-\lambda)^{n-1}=0$$
$$\lambda=0$$
$$-\lambda-\frac{\alpha}{p^n}=0$$
$$\lambda=\frac{+\alpha}{p^n} \qquad \rho(G)=\frac{|\alpha|}{p^n}<1 \qquad -1<\frac{\alpha}{p^n}<1$$
$$-p^n<\alpha<p^n$$

3. JACOBI

$A=D-L-U$

$M=D \quad N=L+U$

$$M\begin{pmatrix} p & & \\ & \ddots & \\ & & p \end{pmatrix} \qquad N=\begin{pmatrix} 0 & & -1 \\ -\alpha & \ddots & \vdots \\ \vdots & \ddots & -1 \\ -\alpha & & 0 \end{pmatrix}$$

$$M^{-1}: \begin{pmatrix} p & & \\ & \ddots & \\ & & p \end{pmatrix}\begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}=\begin{pmatrix} 1 \\ 0 \\ \vdots \\ 1 \end{pmatrix} \qquad \begin{pmatrix} 1/p & & \\ & \ddots & \\ & & 1/p \end{pmatrix}$$

$$PX_1=1 \quad X_1=1/p$$
$$PX_2=0$$
$$PX_3=0$$
$$PX_n=0$$
$$PX_n=1 \to 1/p$$

$$M^{-1}N\begin{pmatrix} 1/p & & \\ & \ddots & \\ & & 1/p \end{pmatrix}\begin{pmatrix} 0 & & -1 \\ -\alpha & \ddots & \vdots \\ \vdots & \ddots & -1 \\ -\alpha & & 0 \end{pmatrix}=\begin{pmatrix} 0 & & -1/p \\ -\alpha/p & \ddots & \vdots \\ \vdots & \ddots & 0 \\ -\alpha/p & & 0 \end{pmatrix}$$

$$\det(AI-S)=\begin{pmatrix} \lambda & & & +1/p \\ \alpha/p & \ddots & \iddots & \vdots \\ \vdots & \ddots & \lambda & 1/p \\ \alpha/p & & & \lambda \end{pmatrix}-$$

$$\lambda\det\begin{pmatrix} \lambda & & +1/p \\ \alpha/p & \ddots & \vdots \\ \alpha/p & & \lambda \end{pmatrix}^{n-1}$$

$$\lambda\lambda\det\begin{pmatrix} \lambda & & \\ & \ddots & \\ & & \lambda \end{pmatrix}$$

$$\lambda^{(n-1)}\det\begin{pmatrix} \lambda & 1/p \\ \alpha/p & \lambda \end{pmatrix}$$

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ESERCIZIO 1:

$f(x) = x + 1 - e^{x/\rho} = 0$

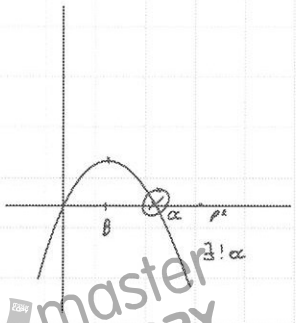
1. $f \in C^\infty(\mathbb{R})$

$\lim_{x \to +\infty} f(x) = -\infty$

$\lim_{x \to -\infty} f(x) = -\infty$

$f'(x) = 1 - \frac{e^{x/\rho}}{\rho}$ $\quad f(x) > 0 \quad x \leq \beta$

$f''(x) > 0 \quad \forall x \in \mathbb{R}$



$f(0) = 0$

2. $x_0 = \rho^2$

$f(\rho^2) = \rho + 1 - e^{\rho} < 0 \qquad f'(\rho^2) < 0 \qquad \rho^2 > \beta$

$x_0 > \alpha$ converge x Teorema di convergenza in corpo $\quad f'(x) \neq 0$, $f(x) < 0$, $f''(x) < 0$

$\beta < x_0 < \alpha \qquad x_0 - \alpha < 0$

$x_1 - \alpha = g(x_0) - g(\alpha) = f'(\xi)(x_0 - \alpha) = \frac{f''(\xi) f(\xi)}{f'(\xi)} (x_0 - \alpha)$ ($f'(\xi) > 0$, $f(\xi) > 0$, $f''(\xi) < 0$)

$x_1 - \alpha > 0 \quad x_1 > 0$

$x_0 = \rho^2 > \beta$ converge

3. function [xk, it] = Newton (Tol, ρ, x_0)

```
f_alpha(x) = x + 1 - exp(x/ρ);
f = @(x) 1 - exp(x-ρ)/ρ;
X0 = ρ^2;

err = inf;
it = 0
while (err >= Tol)

  x new = x old - (f(x old)/f'(x old));

  err = abs(x new - x old);

  x old = x new;

  it = it + 1

end
end
```

16/07/2029

Esercizio 1:

1.)

$p(x) = 0$ con $p(x) = x - \frac{1}{2}\cos(x) - 1$

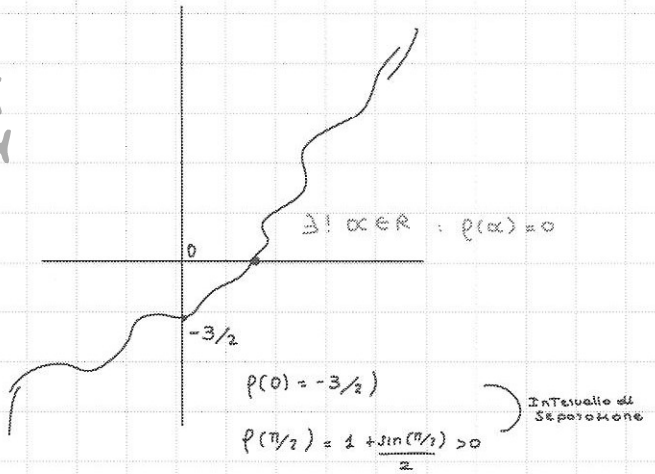
C.E.: C^∞

$\lim_{x \to +\infty} x - \frac{1}{2}\cos x - 1 = +\infty$

$\lim_{x \to -\infty} x - \frac{1}{2}\cos x - 1 = -\infty$

$\lim_{x \to 0} x - \frac{\cos(x)}{2} \to -\frac{3}{2}$

$p'(x) = 1 + \frac{1}{2}\sin(x) > 0 \qquad p'(x) > 0 \ \forall x \in \mathbb{R}$



2.)

Metodo di iterazione funzionale $x_{k+1} = q(x_k)$

$q(x) = \frac{1}{2}\cos(x) + 1$, $k \geq 0$ genera succ. convergente ad α $\forall x_0 \in \mathbb{R}$.

↓

TEOREMA DEL PUNTO FISSO $|q'(\alpha)| < 1$??

$q'(\alpha) = \left| -\frac{1}{2}\sin(\alpha) \right| < 1$?

↓

$-1 < \sin(x) < 1 \quad \forall x \in \mathbb{R} \longrightarrow |q'(\alpha)| \leq \frac{1}{2}$, α = PUNTO ATTRATTIVO → il METODO è LOCALMENTE CONVERGENTE in α.

+ $|q'(x)| < 1 \quad \forall x \in \mathbb{R}$

↓

(i.e. contrazione $\forall x_0 \in I_\alpha(\alpha - \rho, \alpha + \rho) \subseteq \mathbb{R}$

3.) in pwx : $x_0 \in \mathbb{R}$ out pwx : x_K, K

$x_{K+1} = q(x_K)$

$|x_{K+1} - x_K| < 10^{-12}$

```
function [ x, K ] = PUNTO FISSO (x0)

q = @(x) 0.5*cos(x)+1

x = x0;
err = inf;
K = 0;
while (err >= 1.0e-12)

k = k+1;
xnew = q(x);
err = abs(xnew-x);
x = xnew;
end
```

x = x_0 ; xnew = $q(x_0)$; $x - x_0$

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