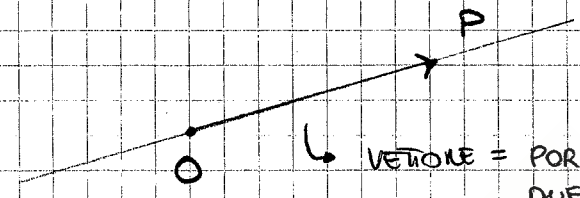
ANALISI II

GEOMETRIA

CONCETTI PRIMITIVI = PUNTO, RETTA, ETC...

DISTANZA EUCLIDEA = DISTANZA MISURATA CON IL RIGHELLO

PRENDIAMO UN PUNTO NEL PIANO A^2

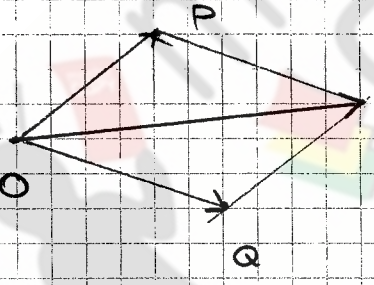


↳ VETTORE = PORZIONE DI RETTA ORIENTATA CHE UNISCE DUE PUNTI

$\vec{OP} = P - O$

SOMMA DI VETTORI

DATI DUE VETTORI v e w



$v + w = \vec{OP} + \vec{OQ} = \vec{OR}$

LA SOMMA DI VETTORI GODE DI QUESTE PROPRIETÀ:

i) $(u+v)+w = u+(v+w)$ → PROPRIETÀ ASSOCIATIVA

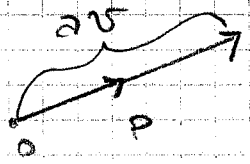
ii) $\vec{OO}$ = VETTORE 0 (ZERO) $v+0 = 0+v = v$ → HA ELEMENTO NEUTRO

iii) DATO v ∃ w TALE CHE $v+w=0$ $(w=-v)$ → HA L'INVERSO

iv) $v+w = w+v$ → PROPRIETÀ COMMUTATIVA

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PRODOTTO PER SCALARE



$a \in \mathbb{R}$

PROPRIETÀ:

v) $a(v+w) = av + aw$ } PROPRIETÀ DISTRIBUTIVA

vi) $(a+b)v = av + bv$ }

vii) $(ab)v = a(bv)$

viii) $0 \cdot \vec{v} = \vec{0}0$ $\quad 1v = v$ $\quad -1v = -v$

- SE LEGHIAMO TUTTE LE PROPRIETÀ DI PRIMA CI ACCORGIAMO CHE HO UNO <u>SPAZIO VETTORIALE</u>

A^2 ASSOCIATO $(V^2, +)$

PIANO GEOMETRICO — SPAZIO VETTORIALE

ESEMPI DI SPAZI VETTORIALI

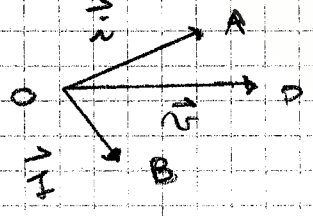
1) $\mathbb{R}$

2) $Pol[x]$ → POLINOMIO DI GRADO d

3) $C^0[a,b]$ → FUNZIONI CONTINUE IN UN INTERVALLO $[a,b]$
(0 = NUMERO DI DERIVATE CONTINUE)

<u>TEOREMA</u>

PRENDIAMO



$\forall v \in V^2 \; \exists! \; a,b \in \mathbb{R}$ t.c. $v = a\vec{i} + b\vec{j}$ $\quad \langle \vec{i}, \vec{j} \rangle$ BASE

LUNGHEZZE NEGLI SPAZI VETTORIALI

$v \in \mathbb{R}^2$ → COPPIE DI NUMERI REALI (x, y) CON $x \in \mathbb{R}$, $y \in \mathbb{R}$

PRODOTTO SCALARE

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$$\langle \cdot, \cdot \rangle \quad \mathbb{R}^2 \times \mathbb{R}^2 \to \mathbb{R}$$

$$\left\langle \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} \right\rangle = x_1 x_2 + y_1 y_2 \to \text{PRODOTTO SCALARE}$$

PROPRIETÀ

i) BILINEARE $\langle v, w \rangle = \langle w, v \rangle$

ii) SIMMETRICO $\langle v+w, z \rangle = \langle v, z \rangle + \langle w, z \rangle$

$\langle \alpha v, z \rangle = \alpha \langle v, z \rangle \quad \forall \alpha \in \mathbb{R} \wedge v, w, z \in \mathbb{R}^2$

PER LE DIMOSTRAZIONI BASTA INSERIRE I VETTORI E SVOLGERE I CALCOLI.

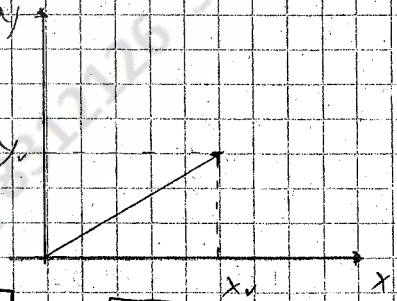
NORMA

$(\mathbb{R}^2, \langle \cdot, \cdot \rangle)$

$\Downarrow$

$(\mathbb{R}^2, \|\cdot\|)$

$\|\cdot\| : \mathbb{R}^2 \to \mathbb{R}^+$

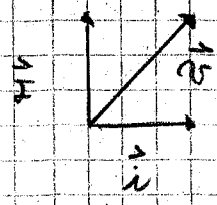


$$\|v\| = \sqrt{\langle v, v \rangle} = \sqrt{\left\langle \begin{pmatrix} x \\ y \end{pmatrix}, \begin{pmatrix} x \\ y \end{pmatrix} \right\rangle} = \sqrt{x^2 + y^2}$$

UNA VOLTA TROVATA LA NORMA POSSIAMO INTRODURRE LA DISTANZA

DISTANZA

$$d(v,w) = \|v-w\| = \sqrt{(x_1-x_2)^2 + (y_1-y_2)^2} \quad \text{con } v = \begin{pmatrix} x_1 \\ y_1 \end{pmatrix} \in w = \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}$$



$$\ell(\vec{i}) = 1 \quad \wedge \quad \vec{i} \perp \vec{j} \quad \ell(\vec{j}) = 1$$

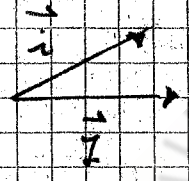
$$\ell(v) = \left\| \begin{pmatrix} x \\ y \end{pmatrix} \right\|$$

DA PIANO GEOMETRICO A SPAZIO VETTORIALE A DEFINIZIONE DI LUNGHEZZA

$$(\mathcal{V}^2, +) \rightsquigarrow (\mathbb{R}^2, d)$$

PARALLELOGRAMMA DI

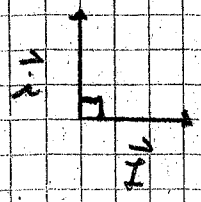
$$(\mathcal{V}^2, +) \rightsquigarrow (\mathbb{R}^2, +)$$



$$\vec{i} = (1,0)$$
$$\vec{j} = (0,1)$$

$v = x\vec{i} + y\vec{j}$ → ISOMORFISMO

$$(\mathcal{V}^2, d) \xrightarrow{\sim} (\mathbb{R}^2, d)$$



$$v = x_1\vec{i} + y_1\vec{j}$$
$$w = x_2\vec{i} + y_2\vec{j}$$

$|\vec{i}| \times |\vec{j}| = 1$ $\qquad d(v,w) = d\left(\begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \begin{pmatrix} x_2 \\ y_2 \end{pmatrix}\right)$ $\qquad i \perp j$

ISOMETRIA

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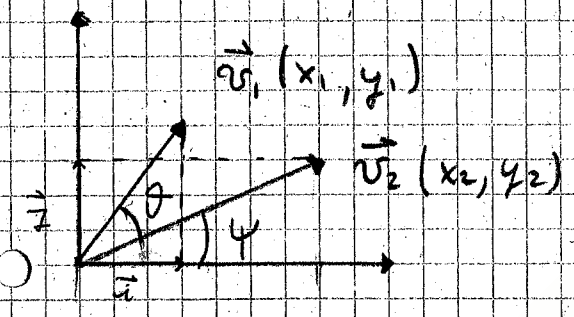
DIMOSTRO CHE CON PRODOTTO SCALARE AMMETTO ANCHE ANGOLI

$$\left(\mathbb{V}^2, \langle\ ,\ \rangle\right)$$

$$\langle v, w\rangle = \left\langle \begin{pmatrix} x_1 \\ y_1 \end{pmatrix}, \begin{pmatrix} x_2 \\ y_2 \end{pmatrix} \right\rangle$$

$v = x_1 i + y_1 j$ $\qquad \|v\| = \sqrt{\langle v, v\rangle}$

$w = x_2 i + y_2 j$ $\qquad d(v, w) = \|v - w\|$



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$$\cos\vartheta = \frac{\langle v_1, v_2\rangle}{\|v_1\| \cdot \|v_2\|}$$

$$\begin{cases} x_1 = \|v_1\| \cdot \cos(\vartheta + \psi) \\ y_1 = \|v_1\| \cdot \operatorname{sen}(\vartheta + \psi) \\ x_2 = \|v_2\| \cos(\psi) \\ y_2 = \|v_2\| \operatorname{sen}(\psi) \end{cases}$$

$$\cos(\vartheta + \psi) = \cos\psi\cos\vartheta - \operatorname{sen}\psi\operatorname{sen}\vartheta$$

$$\operatorname{sen}(\vartheta + \psi) = \cos\psi\operatorname{sen}\vartheta + \operatorname{sen}\psi\cos\vartheta$$

$$x_1 = \|v_1\| \cos\psi\cos\vartheta - \|v_1\| \operatorname{sen}\psi\operatorname{sen}\vartheta$$

$$y_1 = \|v_1\| \cos\psi\operatorname{sen}\vartheta + \|v_1\| \operatorname{sen}\psi\cos\vartheta$$

$$\frac{x_1}{\|v_1\|} = \cos\psi\cos\vartheta - \operatorname{sen}\psi\operatorname{sen}\vartheta$$

$$\frac{y_1}{\|v_1\|} = \cos\psi\operatorname{sen}\vartheta + \operatorname{sen}\psi\cos\vartheta$$

$$\begin{cases} \frac{x_1}{\|v_1\|} = \frac{x_2}{\|v_2\|}\cos\vartheta - \frac{y_2}{\|v_2\|}\operatorname{sen}\vartheta & \text{I} \\ \frac{y_1}{\|v_1\|} = \frac{x_2}{\|v_2\|}\operatorname{sen}\vartheta + \frac{y_2}{\|v_2\|}\cos\vartheta & \text{II} \end{cases}$$

$$\text{I}: \frac{\operatorname{sen}\vartheta}{\|v_2\|} = \frac{x_2\cos\vartheta}{\|v_2\| y_2} - \frac{x_1}{\|v_1\| y_2}$$

$$\text{II}: \frac{y_1}{\|v_1\|} - \frac{x_2\cos\vartheta}{\|v_2\| y_2} = \frac{x_1 x_2}{\|v_1\| y_2} + \frac{y_2^2\cos\vartheta}{\|v_2\|}$$

sostituisco