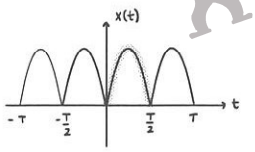
COMPITINO 12/11/2016 FILA A

1) $x(t) = \sum_{n=-\infty}^{+\infty} x_0(t - mT) \qquad x_0(t) = \left| \sin\left(\frac{2\pi t}{T}\right) \right| \text{rect}\left(\frac{t}{T}\right)$

$P_x = ? \qquad X_K = ?$

- $P_x = \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt = \frac{2}{T} \int_{0}^{T/2} \sin^2\left(\frac{2\pi t}{T}\right) dt = \frac{1}{T} \int_{0}^{T/2} 1\, dt - \int_{0}^{T/2} \cos\left(2\,\frac{2\pi t}{T}\right) dt = \frac{1}{T} \cdot \frac{T}{2} = \frac{1}{2}$

$\sin^2 x = \frac{1 - \cos 2x}{2}$




$x(t) = \sum_{n=-\infty}^{+\infty} x_1(t - mT_1)$

$x_1(t) = \left| \text{sen}\left(\frac{2\pi t}{T_1}\right) \right| \text{rect}\left(\frac{t - T_1/4}{T_1/2}\right)$

$T_1 = \frac{T}{2} \qquad f_1 = \frac{2}{T}$

- $X_K = \frac{1}{T_1} \int_{0}^{T_1} x(t)\, e^{-j2\pi K f_1 t} = \frac{1}{T_1} \int_{0}^{T_1} \sin(\pi f_1 t)\, e^{-j2\pi K f_1 t} = \frac{1}{T_1} \frac{1}{2j} \int_{0}^{T_1} \left(e^{j\pi f_1 t} - e^{-j\pi f_1 t}\right) e^{-j2\pi K f_1 t} = \frac{1}{2jT_1} \int_{0}^{T_1} e^{j2\pi f_1 t\left(\frac{1}{2}-K\right)} - e^{-j2\pi f_1 t\left(\frac{1}{2}+K\right)}$

$= \frac{1}{2jT_1} \left[ \frac{e^{j2\pi f_1 t\left(\frac{1}{2}-K\right)}}{j2\pi f_1\left(\frac{1}{2}-K\right)} \Bigg|_0^{T_1} - \frac{e^{j2\pi f_1 t\left(\frac{1}{2}+K\right)}}{j2\pi f_1\left(\frac{1}{2}+K\right)} \Bigg|_0^{T_1} \right] = \frac{1}{2j\cancel{T_1}} \left[ \frac{\overbrace{e^{j2\pi\left(\frac{1}{2}-K\right)}}^{-1\,(*)} - 1}{j2\pi\cancel{f_1}\left(\frac{1}{2}-K\right)} - \frac{\overbrace{e^{j2\pi\left(\frac{1}{2}+K\right)}}^{+1} - 1}{j2\pi\cancel{f_1}\left(\frac{1}{2}+K\right)} \right] = \frac{1}{2\pi\left(\frac{1}{2}-K\right)} + \frac{1}{2\pi\left(\frac{1}{2}+K\right)} =$

$= \frac{1}{2\pi\left(\frac{1}{4} - K^2\right)}$

$(*) = e^{j\pi} \cdot e^{-2j\pi K}$ ($e^{j\pi} = -1$, $e^{-2j\pi K} = 1$)

2) $H(f) = \frac{j2\pi F}{1 + j2\pi F}$

$h(t) = ?$

$y(t) = ? \qquad x(t) = e^{-t} u(t)$

- $H(f) = \frac{1 + j2\pi F - 1}{1 + j2\pi F} = 1 - \frac{1}{1 + j2\pi F}$

$h(t) = \delta(t) - e^{-t} u(t)$

- $X(f) = \frac{1}{1 + j2\pi F}$

$Y(f) = H(f) X(f) = \frac{1}{1 + j2\pi F} - \left(\frac{1}{1 + j2\pi F}\right)^2$

$y(t) = e^{-t} u(t) - t\, e^{-t} u(t) = e^{-t} u(t)(1 - t)$

COMPITINO 12/11/2016 FILA B

1) $x(t) = \sum_{n=-\infty}^{+\infty} x_0(t - nT) \qquad x_0(t) = t\,\text{rect}\left(\frac{t}{T}\right)$

$P_x = ? \qquad X_K = ?$

- $P_x = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{T_0/2} |x(t)|^2 dt = \frac{1}{T_0}\int_{-\frac{T_0}{2}}^{T_0/2} t^2 dt = \frac{1}{T_0}\left[\frac{t^3}{3}\right]_{-\frac{T_0}{2}}^{\frac{T_0}{2}} = \frac{1}{T_0}\left[\frac{T_0^3}{8}\cdot\frac{1}{3} + \frac{T_0^3}{8}\cdot\frac{1}{3}\right] = \frac{2T_0^2}{24} = \frac{T_0^2}{12}$

- $X_K = \frac{1}{T}\int_{-\frac{T}{2}}^{T/2} x(t)\, e^{-j2\pi K f_0 t} dt = \frac{1}{T}\int_{-T/2}^{T/2} t\, e^{-j2\pi K f_0 t} dt = \frac{1}{T}\left[\int_{-T/2}^{T/2} t\cos(2\pi K f_0 t)\,dt - j\int_{-T/2}^{T/2} t\,\text{sen}(2\pi K f_0 t)\,dt\right] = -\frac{2j}{T}\int_0^{T/2} t\,\text{sen}(2\pi K f_0 t)\,dt$ (=)

$y = 2\pi K f_0 t$

$t = \frac{y}{2\pi f_0 K} \qquad dt = \frac{dy}{2\pi f_0 K}$

(=) $-\frac{2j}{T}\int_0^{\pi K} \frac{y}{2\pi K f_0}\,\text{sen}(y)\,\frac{dy}{2\pi K f_0} = -\frac{j}{2\pi^2 K^2 f_0}\int_0^{\pi K} y\,\text{sen}\,y\,dy = -\frac{j}{2\pi^2 K^2 f_0}\left[\text{sen}\,y - y\cos y\right]_0^{\pi K} = -\frac{j}{2\pi^2 K^2 f_0}\left[-(-1)^K\cdot K\pi\right]$

$\int y\,\text{sen}\,y = -y\cos y + \text{sen}\,y$

$= \frac{j(-1)^K}{2\pi K f_0} \qquad K \neq 0$

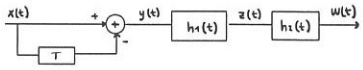
$X_0 = \int x(t)dt = 0$

2) $x(t) = \left[1 - \cos\left(\frac{\pi t}{4T}\right)\right] \qquad f_0 = \frac{1}{8T}$

$h_1(t) = \delta(t-T) + \delta(t+T)$

$h_2(t) = 4\,\text{rect}\left(\frac{t}{2T}\right)\cos\left(\frac{3\pi t}{4T}\right)$ $\quad f_0 = \frac{3}{8}$

$Z(f) = ? \qquad W(f) = ?$




$y(t) = x(t) - x(t-T)$

$z(t) = y(t) \circledast h_1(t)$

$w(t) = z(t) \circledast h_2(t)$

$z(t) = y(t) \circledast h_1(t)$

$z(t) = x(t-T) + x(t+T) - x(t-2T) - x(T)$

$Z(f) = X(f)e^{-j2\pi fT} + X(f)e^{j2\pi fT} - X(f)e^{-j4\pi fT} - X(f) = X(f)\left[e^{-j2\pi fT} + e^{j2\pi fT}\right] - X(f)\left[e^{-j4\pi fT} + 1\right] =$

$= 2X(f)\cos(2\pi fT) - X(f)e^{-j2\pi fT}\left[e^{-j2\pi fT} + e^{j2\pi fT}\right] = 2X(f)\cos(2\pi fT) - 2X(f)\cos(2\pi fT)\,e^{-j2\pi fT} =$

$= 2X(f)\cos(2\pi fT)\left[1 - e^{-j2\pi fT}\right] = 4jX(f)\,e^{-j\pi fT}\cos(2\pi fT)\,\text{sen}(\pi fT)$

$X(f) = \mathcal{F}^{-1}[x(t)] = \delta(f) - \frac{1}{2}\delta\left(f - \frac{1}{8T}\right) - \frac{1}{2}\delta\left(f + \frac{1}{8T}\right)$

$H_2(f) = \frac{W(f)}{Z(f)} \Rightarrow W(f) = H_2(f) Z(f)$

$Z(f)$ la ho

$H_2(f) = 4T \operatorname{sinc}\left(2\left(f - \frac{3}{8T}\right)T\right) + 4T \operatorname{sinc}\left(2\left(f + \frac{3}{8T}\right)T\right)$

$W(f) = Z(f) H_2(f)$

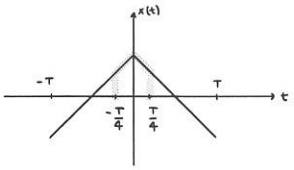
COMPITINO 12/11/2016 FILA C

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1) $x(t) = \sum_{n=-\infty}^{+\infty} x_0(t - mT) \qquad x_0(t) = \left(1 - \frac{2|t|}{T}\right) \operatorname{rect}\left(\frac{2t}{T}\right)$

$P_x = ? \quad X_k = ?$




- $P_x = \frac{1}{T} \int_{-T/4}^{T/4} |x(t)|^2 dt = \frac{2}{T} \int_0^{T/4} \left(1 - \frac{2t}{T}\right)^2 dt = \frac{2}{T} \int_0^{T/4} \left(1 + \frac{4t^2}{T^2} - \frac{4t}{T}\right) dt = \frac{2}{T}\frac{T}{4} + \frac{2}{T}\left[\frac{4t^3}{3T^2} - \frac{2t^2}{T}\right]_0^{T/4} = \frac{1}{2} + \frac{2}{T}\left[\frac{T}{16 \cdot 3} - \frac{T}{8}\right] =$

$= \frac{1}{2} + \frac{1}{24} - \frac{1}{4} = \frac{12 + 1 - 6}{24} = \frac{7}{24}$

- $X_k = \frac{1}{T} \int_{-T/4}^{T/4} \left(1 - \frac{2|t|}{T}\right) e^{-j2\pi k f_0 t} dt = \frac{2}{T} \int_0^{T/4} \left(1 - \frac{2t}{T}\right) \cos(2\pi k f_0 t) dt = \underbrace{\frac{2}{T} \int_0^{T/4} \cos(2\pi k f_0 t) dt}_{1} - \underbrace{\frac{4}{T^2} \int_0^{T/4} t \cos(2\pi k f_0 t) dt}_{2}$ (=)

1) $\frac{2}{T} \int_0^{T/4} \cos(2\pi k f_0 t) dt = \frac{2}{T} \left[\frac{\operatorname{sen}(2\pi k f_0 t)}{2\pi k f_0}\right]_0^{T/4} = \frac{2}{T} \frac{\operatorname{sen}\left(\frac{\pi k}{2}\right)}{2\pi k f_0} = \frac{\operatorname{sen}\left(\frac{\pi k}{2}\right)}{\pi k} = \frac{1}{2} \operatorname{sinc}\left(\frac{k}{2}\right)$

2) $\frac{4}{T^2} \int_0^{T/4} t \cos(2\pi k f_0 t) dt = \Big| \begin{matrix} y = 2k\pi f_0 t \\ t = \frac{y}{2k\pi f_0} \end{matrix} = \frac{4}{T^2} \int_0^{\frac{k\pi}{2}} \frac{y}{2\pi k f_0} \cos y \frac{dy}{2\pi k f_0} = \frac{1}{\pi^2 k^2} \int_0^{\frac{k\pi}{2}} y \cos y \, dy = \Big| \begin{matrix} \\ \int y\cos y = y\operatorname{sen} y + \cos y \end{matrix} = \frac{1}{\pi^2 k^2} \left[y \operatorname{sen} y + \cos y\right]_0^{\frac{k\pi}{2}} =$

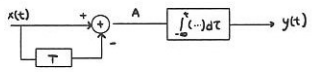
$= \frac{1}{\pi^2 k^2} \left[\frac{k\pi}{2} \operatorname{sen}\left(\frac{k\pi}{2}\right) + \cos\left(\frac{k\pi}{2}\right) - 1\right] = \frac{\operatorname{sen}\left(\frac{k\pi}{2}\right)}{2\pi k} - \frac{1}{8} \frac{\operatorname{sen}^2\left(\frac{k\pi}{4}\right)}{\frac{\pi^2 k^2}{16}} = \frac{1}{4} \operatorname{sinc}\left(\frac{k}{2}\right) - \frac{1}{8} \operatorname{sinc}^2\left(\frac{k}{4}\right)$

(=) $\frac{1}{2} \operatorname{sinc}\left(\frac{k}{2}\right) - \frac{1}{4} \operatorname{sinc}\left(\frac{k}{2}\right) + \frac{1}{8} \operatorname{sinc}^2\left(\frac{k}{4}\right) = \frac{1}{4} \operatorname{sinc}\left(\frac{k}{2}\right) + \frac{1}{8} \operatorname{sinc}^2\left(\frac{k}{4}\right)$

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2) $x(t) = 4 \operatorname{rect}\left(\frac{t - T/2}{T}\right)$

$y(t) = ? \quad Y(f) = ?$



$$x_A(t) = x(t) - x(t-T)$$

$$y(t) = \int_{-\infty}^{t} x_A(\tau)\, d\tau$$

$$\begin{cases} X_A(f) = X(f) - X(f)e^{-j2\pi fT} = X(f)\left(1 - e^{-j2\pi fT}\right) = X(f)\, e^{-j\pi fT}\, 2j \operatorname{sen}(\pi fT) \\ X(f) = 4T \operatorname{sinc}(fT)\, e^{-j\pi fT} \end{cases}$$

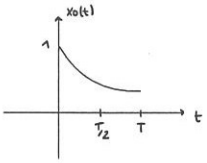
$$Y(f) = \frac{X_A(f)}{j2\pi f} = \frac{X(f)}{\pi f} e^{-j\pi fT} \operatorname{sen}(\pi fT) = 4T^2 \operatorname{sinc}^2(fT)\, e^{-j2\pi fT}$$

$$y(t) = 4T\, tr\left(\frac{t - T}{2T}\right)$$

COMPITINO 12/11/2016 FILA D

1) $x(t) = \sum_{n=-\infty}^{+\infty} x_0(t - nT) \qquad x_0(t) = e^{-t}\, rect\left(\frac{t - T/2}{T}\right)$

$P_x = ? \quad x_K = ?$




- $P_x = \frac{1}{T}\int_0^T e^{-2t} = \frac{1}{T}\frac{e^{-2t}}{-2}\Big|_0^T = \frac{1}{T}\,\frac{e^{-2T} - 1}{-2} = \frac{1 - e^{-2T}}{2T}$

- $x_K = \frac{1}{T}\int_0^T e^{-t} e^{-j2\pi Kf_0 t} = \frac{1}{T}\int_0^T e^{-(1+j2\pi Kf_0)t} = \frac{1}{T}\,\frac{e^{-(1+j2\pi Kf_0)t}}{-(1+j2\pi Kf_0)}\Big|_0^T = \frac{1}{T}\,\frac{e^{-(1+j2\pi Kf_0)T} - 1}{-(1+j2\pi Kf_0)} =$

$$\frac{1}{T}\,\frac{e^{-T}\cdot \overbrace{e^{-j2\pi K}}^{1} - 1}{-(1+j2\pi Kf_0)} = \frac{1 - e^{-T}}{T + j2\pi K}$$

2) $h_1(t) = e^{-\alpha t}u(t)$

$h_2(t) = e^{-\beta t}u(t) \qquad \alpha, \beta > 0 \qquad \alpha \neq \beta$

$h(t) = ? \quad H(f) = ?$ in serie e poi in parallelo

- SERIE




$$y(t) = x(t) \circledast h_{eq}$$

$$h_{eq} = h_1 \circledast h_2$$

$$\begin{cases} H_{eq} = H_1 H_2 \\ H_1 = \frac{1}{\alpha + j2\pi f} \qquad H_2 = \frac{1}{\beta + j2\pi f} \end{cases}$$

$$H_{eq} = \frac{1}{(\alpha + j2\pi f)(\beta + j2\pi f)}$$

$$|H_{eq}| = \frac{1}{\sqrt{(\alpha^2 + 4\pi^2 f^2)(\beta^2 + 4\pi^2 f^2)}}$$

$$\angle H_{eq} = -\operatorname{arctg}\frac{2\pi f}{\alpha} - \operatorname{arctg}\frac{2\pi f}{\beta}$$