$$P_0(f) = \frac{T}{2}\,\mathrm{sinc}\left(f\frac{T}{2}\right)\left(e^{-j\frac{\pi}{2}fT} - e^{-j\frac{3\pi}{2}fT}\right)$$

$$P(f) = \sum_{K=-\infty}^{+\infty} P_K\,\delta\left(f - \frac{K}{T_0}\right)$$

$$P_K = \frac{1}{T}P_0\left(\frac{K}{T}\right) = \frac{1}{2}\,\mathrm{sinc}\left(\frac{K}{2}\right)e^{-j\frac{\pi}{2}K} - \frac{1}{2}\,\mathrm{sinc}\left(\frac{K}{2}\right)e^{+j\frac{3}{2}\pi K}$$

$$= -j\,\mathrm{sinc}\left(\frac{K}{2}\right)\mathrm{seu}\left(K\frac{\pi}{2}\right)$$

$$P(f) = \sum_{K=-\infty}^{+\infty} -j\,\mathrm{sinc}\left(\frac{K}{2}\right)\mathrm{seu}\left(K\frac{\pi}{2}\right)\delta\left(f - \frac{K}{T}\right)$$

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$$Z(f) = X(f) \circledast P(f)$$

$$X(f) = \mathrm{rect}(fT) = \mathrm{rect}\left(\frac{f}{\frac{1}{T}}\right)$$

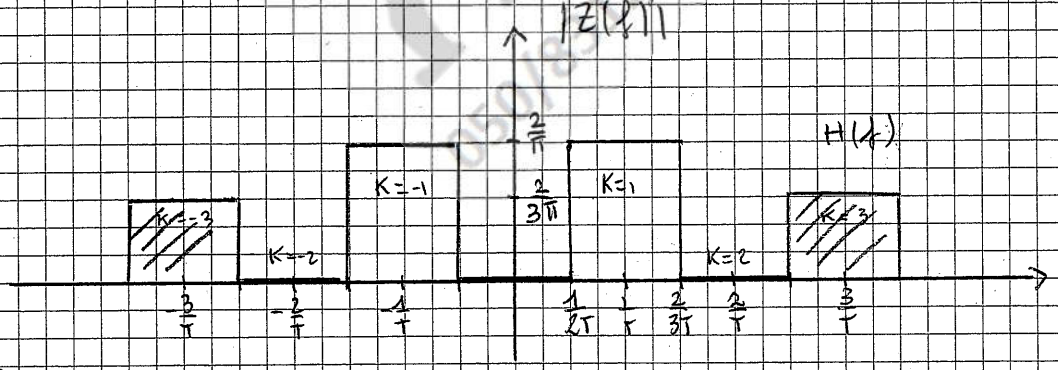
$$Z(f) = \sum_{K=-\infty}^{+\infty} -j\,\mathrm{sinc}\left(\frac{K}{2}\right)\mathrm{seu}\left(K\frac{\pi}{2}\right)\mathrm{rect}\left(\frac{f-\frac{K}{T}}{\frac{1}{T}}\right)$$

$K=0 \quad P_K = 0$

$K=1 \quad P_K = -j\frac{2}{\pi}$ $\qquad K=-1 \quad P_K = j\frac{2}{\pi}$

$K=2 \quad P_K = 0$ $\qquad K=-2 \quad P_K = 0$

$K=3 \quad P_K = -j\frac{2}{3\pi}$ $\qquad K=-3 \quad P_K = j\frac{2}{3\pi}$



$$Y(f) = -j\frac{2}{3\pi}\,\mathrm{rect}\left(\frac{f-\frac{3}{T}}{\frac{1}{T}}\right) + j\frac{2}{3\pi}\,\mathrm{rect}\left(\frac{f+\frac{3}{T}}{\frac{1}{T}}\right)$$

$$y(t) = -j\frac{2}{3\pi T}\,\mathrm{sinc}\left(\frac{t}{T}\right)e^{-j6\pi\frac{t}{T}} + j\frac{2}{3\pi T}\,\mathrm{sinc}\left(\frac{t}{T}\right)e^{j6\pi\frac{t}{T}} =$$

$$= \frac{4}{3\pi T}\,\mathrm{sinc}\left(\frac{t}{T}\right)\mathrm{seu}\left(6\pi\frac{t}{T}\right)$$

ESAME 12/01/2017 FILA (D)

ESERCIZIO (1)

$h_1(t) = e^{-t} u(t)$

$h_2(t) = \frac{1}{T} \text{sinc}\left(\frac{2t}{T}\right) \cos\left(\frac{6\pi t}{T}\right)$

1) $H_{eq}(f) = ?$ grafico $|H_{eq}(f)|$

$X(t) = \text{sinc}\left(\frac{t}{T}\right) c(t)$

$c(t) = 2 \sum_{n=-\infty}^{+\infty} \delta(t - nT)$

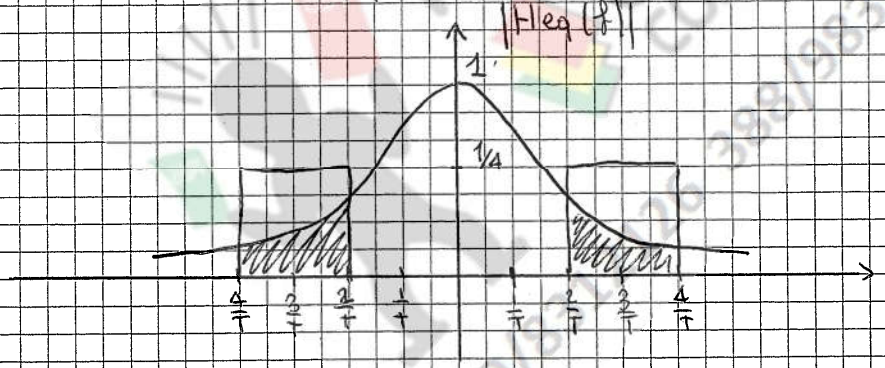
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2) $Y(f) = ?$ $E_y = ?$

SOLUZIONE

$H_1(f) = \frac{1}{1 + j2\pi f}$ $\quad H_1(0) = 1$

$H_2(f) = \frac{1}{4} \text{rect}\left(\frac{f - \frac{3}{T}}{\frac{2}{T}}\right) + \frac{1}{4} \text{rect}\left(\frac{f + \frac{3}{T}}{\frac{2}{T}}\right)$



$Y(f) = H_{eq}(f) \cdot X(f)$

$H_{eq}(f) = \frac{1}{1 + j2\pi f} \left[\frac{1}{4} \text{rect}\left(\frac{f - \frac{3}{T}}{\frac{2}{T}}\right) + \frac{1}{4} \text{rect}\left(\frac{f + \frac{3}{T}}{\frac{2}{T}}\right) \right]$

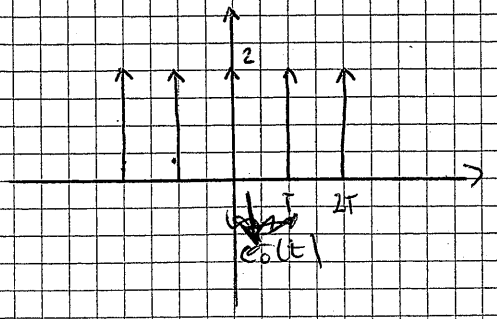
$c(t) = 2 \sum_{n=-\infty}^{+\infty} \delta(t - nT)$

$n = 0 \quad c(t) = 2\delta(t)$

$n = 1 \quad c(t) = 2\delta(t - T)$

$n = -1 \quad c(t) = 2\delta(t + T)$

$n = 2 \quad c(t) = 2\delta(t - 2T)$



$$c_0(t) = 2\delta(t) + 2\delta(t-T)$$

$$C_0(f) = 2 + 2e^{-j2\pi fT} = 2e^{-j\pi fT}\left(e^{j\pi fT} + e^{-j\pi fT}\right) =$$

$$= 4e^{-j\pi fT}\cos(\pi fT)$$

$$C_k = \frac{1}{T}\,4\,e^{-j\pi k}\cos(\pi k)$$

$$C(f) = \sum_{k=-\infty}^{+\infty} \frac{1}{T}\,4\,e^{-j\pi k}\cos(\pi k)\,\delta\left(f - \frac{k}{T}\right)$$

$$X(f) = T\,\mathrm{rect}(fT) \circledast C(f)$$

$$c_0(t) = 2\delta(t) \qquad C_0(f) = 2 \qquad C_k = \frac{2}{T}$$

$$X(f) = T\,\mathrm{rect}(fT) \circledast \sum_{k=-\infty}^{+\infty} \frac{2}{T}\,\delta\left(f - \frac{k}{T}\right) =$$

$$= \sum_{k=-\infty}^{+\infty} 2\,\mathrm{rect}\left(\frac{f - \frac{k}{T}}{1/T}\right)$$

$k=0$ Nello finire $X(f) = 2$

$$Y(f) = 2\,H_{eq}(f)$$

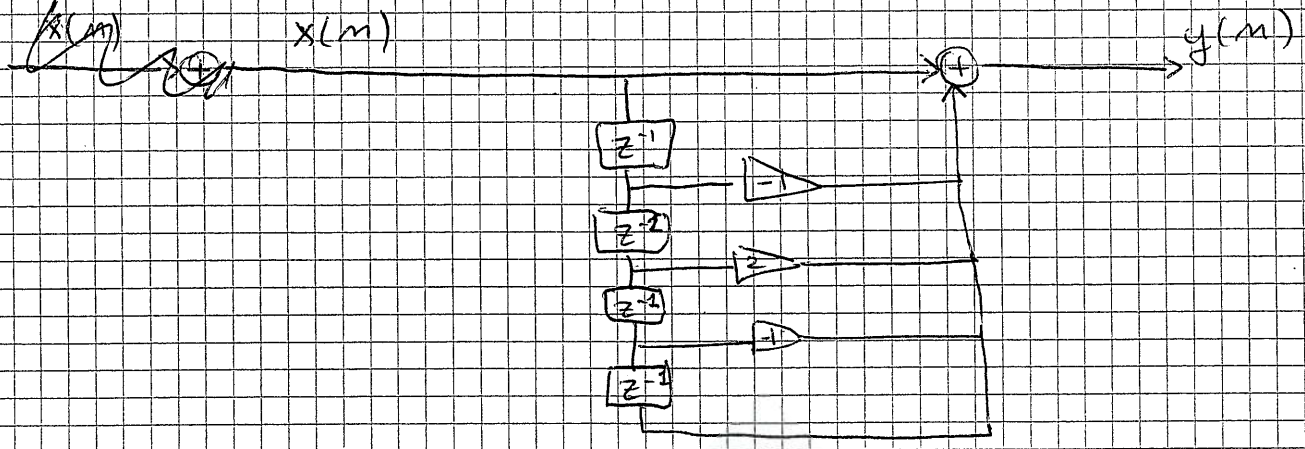
$$Y(f) = \frac{1}{1 + j2\pi f}\left[\frac{1}{4}\,\mathrm{rect}\left(\frac{f - \frac{3}{T}}{\frac{2}{T}}\right) + \frac{1}{4}\,\mathrm{rect}\left(\frac{f + \frac{3}{T}}{2/T}\right)\right]$$

$$E_y = 2\int_{-\infty}^{+\infty} \frac{1}{1 + 4\pi^2 f^2} \cdot \frac{1}{4}\,\mathrm{rect}\left(\frac{f - \frac{3}{T}}{2/T}\right) df =$$

$$= 2 \cdot \frac{1}{4_2} \int_{2/T}^{4/T} \frac{1}{1 + 4\pi^2 f^2}\,df = \frac{1}{4\pi}\left[\mathrm{arctg}\left(\frac{8\pi}{T}\right) - \mathrm{arctg}\left(\frac{4\pi}{T}\right)\right]$$

ESERCIZIO 2

$$y(n) = x(n) - x(n-1) + 2x(n-2) - x(n-3) + x(n-4)$$



$$Y(z) = X(z) - z^{-1}X(z) + 2z^{-2}X(z) - z^{-3}X(z) + z^{-4}X(z)$$

$$H(z) = 1 - z^{-1} + 2z^{-2} - z^{-3} + z^{-4} =$$

$$= \frac{z^4 - z^3 + 2z^2 - z + 1}{z^4}$$

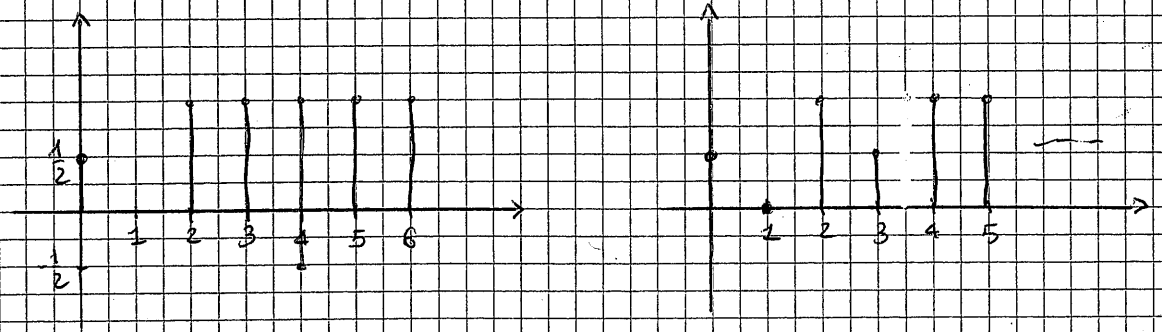
$$x(n) = u(n) \qquad X(z) = \frac{z}{z-1} = \frac{1}{1-z^{-1}}$$

$$Y(z) = \frac{z(z^4 - z^3 + 2z^2 - z + 1)}{z^3(z-1)} =$$

$$= \frac{z^4 - z^3 + 2z^2 - z + 1}{z^4 - z^3} = 1 + \frac{2z^2 - z + 1}{z^4 - z^3} =$$

$$= 1 + \frac{2z^{-2}}{1 - z^{-1}} - \frac{1}{z^3} = \frac{1}{2} + \frac{1}{z^2 - z} + \frac{1}{2z^3}$$

$$y(n) = \frac{1}{2}\delta(n) + u(n-2) - \frac{1}{2}\delta(n-3)$$



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$\bar{H}(f)$ grafico $|H(f)|$

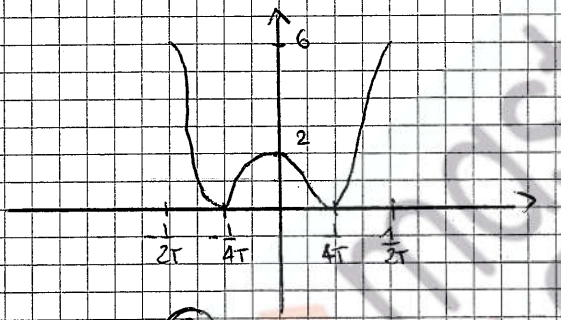
$z = e^{j2\pi f T}$

$$\bar{H}(f) = 2e^{-j4\pi fT}\left[\cos(4\pi fT) + 2\,\mathrm{sen}^2(\pi fT)\right]$$

$$|\bar{H}(f)| = 2\,|\cos(4\pi fT)| + 4\,|\mathrm{sen}^2(\pi fT)|$$

$$|\bar{H}(0)| = 2(1+0) = 2$$

$$|H(\pm\tfrac{1}{2T})| = 6 \qquad |H(\pm\tfrac{1}{4T})| = \emptyset$$



ESERCIZIO ③

$$y(t) = \sqrt{|x(t)+2|}$$

$$y(t) = y_1(t) + y_2(t) = \sqrt{|x_1(t)+2|} + \sqrt{|x_2(t)+2|}$$

$$x(t) = x_1(t) + x_2(t)$$

$$z(t) = \sqrt{|x_1(t)+x_2(t)+2|}$$ NO LINEARE

TEMPO INV.

$y(t) = \mathcal{T}[x(t)]$ $\quad z(t) = \mathcal{T}[x(t-t_0)]$

$y(t-t_0) = \mathcal{T}[x(t-t_0)]$

$$z(t) = \sqrt{|x(t-t_0)+2|}$$

$$y(t-t_0) = \sqrt{|x(t-t_0)+2|}$$

SI $T=1$

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