TCF

ESERCIZIO 1

$y(t) = x(t) \otimes h(t)$ $\qquad x(t) = e^{-t} u(t)$

$h(t) = \text{rect}\left(\frac{t-3}{T}\right)$

$Y(f) = X(f) \cdot h(f)$

$x(t) = e^{-t} u(t) \Rightarrow X(f) = \frac{1}{1 + j2\pi f}$

$h(t) = \text{rect}\left(\frac{t-3}{T}\right) \Rightarrow h(f) = T \,\text{sinc}(fT)\, e^{-j2\pi f \cdot 3}$

$Y(f) = \frac{T \,\text{sinc}(fT)\, e^{-j6\pi f}}{1 + j2\pi f}$

? Calcolare modulo e fase di $Y(f)$

$|Y(f)| = \frac{T \cdot |\text{sinc}(fT)| \cdot 1}{\sqrt{1 + 4\pi^2 f^2}}$

$\angle Y(f) = \angle \text{sinc}(fT) - 6\pi f - \text{arctg}(2\pi f)$

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ESERCIZIO 2

$$x_2(t) = \frac{d}{dt}\left(e^{-2t}\sin(t)\cdot u(t)\right)$$

$$y(t) = e^{-2t}u(t) \Rightarrow Y(f) = \frac{1}{2+j2\pi f}$$

$$X_2(f) = 2\pi j f \cdot Z(f)$$

$$z(t) = y(t)\cdot \sin(t) \Rightarrow Z(f) = \frac{1}{2j}\left[Y(f-f_0) - Y(f+f_0)\right]$$

Applico la proprietà di modulazione:

$$Z(f) = \frac{1}{2j}\left[\frac{1}{2+j2\pi(f-1/2\pi)} - \frac{1}{2+j2\pi(f+1/2\pi)}\right]$$

$$t_0 = 2\pi \Rightarrow f_0 = \frac{1}{t_0} = \frac{1}{2\pi}$$

$$X_2(f) = \frac{2\pi f}{2j}\left[\frac{1}{2+j2\pi(f-1/2\pi)} - \frac{1}{2+j2\pi(f+1/2\pi)}\right]$$

$$X_2(f) = \pi f\left[\frac{1}{2+j2\pi(f-1/2\pi)} - \frac{1}{2+j2\pi(f+1/2\pi)}\right]$$

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ESERCIZIO 3

$$x(t) = 2\,\mathrm{sinc}(t)\cdot \mathrm{sinc}(2t) \Rightarrow x(t) = 2\,y_1(t)\cdot y_2(t)$$

$$y_1(t) = \mathrm{sinc}(t)$$

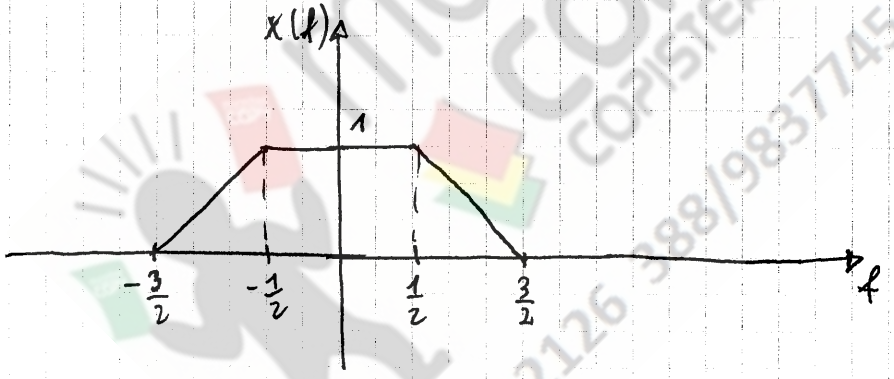
$$y_2(t) = \mathrm{sinc}(2t)$$

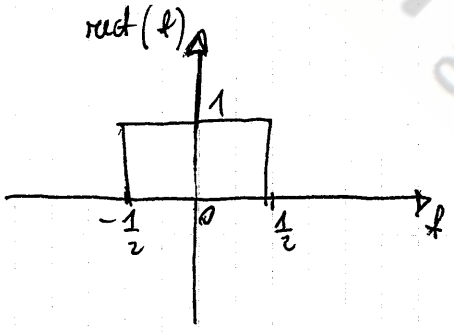
$$X(f) = 2\,Y_1(f) \otimes Y_2(f)$$

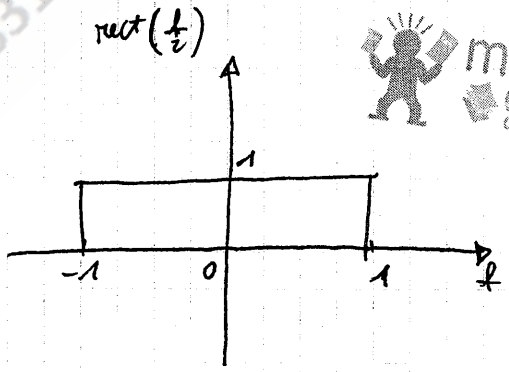
$$Y_1(f) = \mathrm{rect}\left(\frac{f}{1}\right)$$

$$Y_2(f) = \frac{1}{2}\,\mathrm{rect}\left(\frac{f}{2}\right)$$

$$X(f) = \frac{2}{2}\,\mathrm{rect}(f) \otimes \mathrm{rect}\left(\frac{f}{2}\right)$$







- ESTREMO INFERIORE CONVOLUZIONE: $\quad min = -\frac{1}{2} - 1 = -\frac{3}{2}$

- ESTREMO SUPERIORE CONVOLUZIONE: $\quad max = \frac{1}{2} + 1 = \frac{3}{2}$

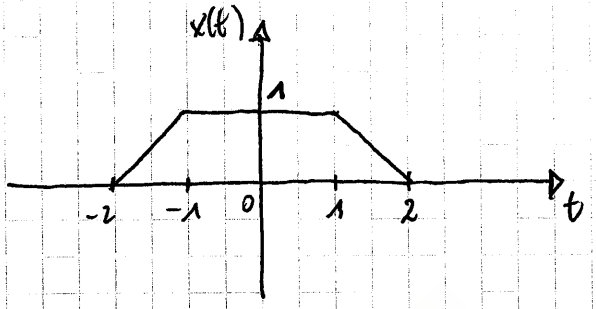
- VALORE INTERMEDIO: $\quad x_1 = -\frac{1}{2} - (-1) = \frac{1}{2}$

$$x_2 = \frac{1}{2} - (1) = -\frac{1}{2}$$

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ESERCIZIO 4

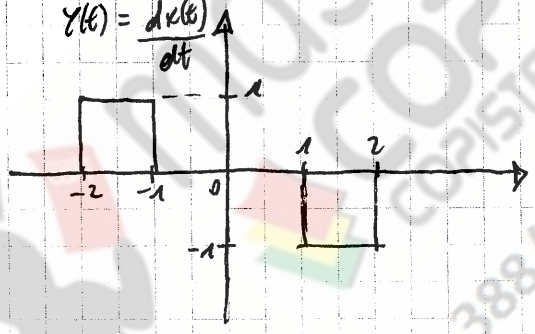
$$x(t) = \begin{cases} t+2 & -2 < t < -1 \\ 1 & -1 < t < 1 \\ -t+2 & 1 < t < 2 \\ 0 & \text{altrove} \end{cases}$$



Applico il teorema di integrazione:

$$Y(0) = 0 \Rightarrow X(f) = \frac{Y(f)}{j\,2\pi f}$$

$$y(t) = \frac{dx(t)}{dt}$$



$$Y(f) = \text{rect}\left(\frac{t+3/2}{1}\right) - \text{rect}\left(\frac{t-3/2}{1}\right)$$

$$Y(f) = \text{sinc}(f)\, e^{j2\pi f \frac{3}{2}} - \text{sinc}(f)\, e^{-j2\pi f \cdot \frac{3}{2}}$$

$$Y(f) = \text{sinc}(f)\left(\frac{e^{j\pi f 3} - e^{-j\pi f 3}}{2j}\right) \cdot 2j = 2j \cdot \text{sinc}(f) \cdot \sin(3\pi f)$$

$\sin(3\pi f)$

$$X(f) = \frac{Y(f)}{j2\pi f} = \frac{\text{sinc}(f) \cdot \sin(3\pi f)}{3\pi f} \cdot 3 = 3\,\text{sinc}(f) \cdot \text{sinc}(3f)$$

$\text{sinc}(3f)$

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ESERCIZIO 5

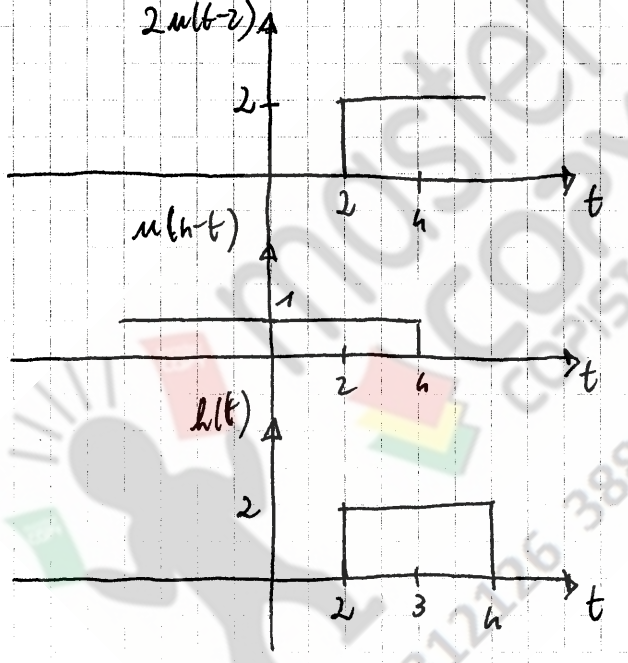
$$y(t) = x(t) \otimes h(t)$$

$$x(t) = 5e^{-t}u(t)$$

$$h(t) = 2\,u(t-2)\,u(4-t)$$

$$Y(f) = X(f) \cdot H(f)$$

$$X(f) = \frac{5}{1 + j2\pi f}$$



$$h(t) = 2 \operatorname{rect}\left(\frac{t-3}{2}\right)$$

$$H(f) = 2 \cdot 2\,\operatorname{sinc}(2f)\,e^{-j2\pi f \cdot 3} = 4\operatorname{sinc}(2f)\,e^{-j6\pi f}$$

$$Y(f) = \frac{20\operatorname{sinc}(2f)\,e^{-j6\pi f}}{1 + j2\pi f}$$

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