Pot. Termodinamici:

- funzioni di stato di (p, V, T, N)
- stazionarie all'equilibrio

Equilibrio: microstato che massim. numero microstati: associati. PESO $S+\Delta T$

$U = U(S, V, N)$

1) $dU = T\,dS - p\,dV + \mu\,dN$ — U, $-TS$

2) $dS = \frac{dU}{T} + \frac{p}{T}dV - \frac{\mu}{T}dN$ — F, $+pV$, $-N\mu$, Φ, Ω

3) $dF = -S\,dT - p\,dV + \mu\,dN$ $\quad (F = U - TS)$

4) $d(\Phi) = -S\,dT + V\,dp + \mu\,dN$ $\quad (\Phi = F + pV)$

5) $d\Omega = -S\,dT - p\,dV - N\,d\mu$ $\quad (\Omega = F - N\mu)$

* $\Phi(N) = N\,\phi(1) = N\mu$ $\quad \Omega = -pV$ $\quad (\text{se } p = \mu N)$

$\Phi = N_\mu(p, T)$ ESTENS

$\Omega = -pV$ $\quad F = U - \left(\frac{\partial S}{\partial U}\right)^{-1} S$

$\Omega = \frac{x}{D} U$

Trasf. Legendre, Molt. Lagrange

$S(U,V,N) = k \ln \{$ peso stat. micro in micro $\} = k \ln \Gamma$

$$\Gamma = \int_{V \le H \le V+\Delta V} \prod_{i=1}^{N} \frac{d^3p_i \, d^3q}{h^{3N} N!} = \int d\sigma^N = \Omega^*$$

$1/N!$ che appr. alle e simm scambi

- $\frac{\Delta V}{V} \ll 1$, $\Omega^* = \bar{\Omega}^* \{1 + f(U, \Delta U, V, N)\}$ (Pacc)

$\ln \Omega^* = \ln \bar{\Omega}^* + \ln(1 + f(U, \Delta U))$, $e^{S/k} = \int d\sigma^N g(H)$

- $g(H) = \begin{cases} 1 \text{ se } \Psi \le H \le V+\Delta V \\ 0 \text{ altrim} \end{cases} \sim \begin{cases} 1 \text{ se } H \le V \\ 0 \text{ altrim} \end{cases}$

Se $N \to \infty$, $V_{sfera} \sim V_{cubo}$ [D'Elia] (MICROCANONICO)

- $e^{-F(T,V,N)/kT} = \int d\sigma^N e^{-H(p,q)/kT}$

[CANONICO] (D'Elia) (Manzari)

- $e^{-\Omega(T,V,\mu)/kT} = \sum_{N=0}^{\infty} e^{N\mu/kT} \int d\sigma^N e^{-H/kT}$

$\{H, N\} \Rightarrow ?$ [Se $N \to \infty$ GRANDI N] GRANCANON (Manzari)

- Interaz. reciproche $\times$ ep

↳ debole: $PV = NRT$

$N = \frac{\partial \Omega}{\partial \mu} \Rightarrow \Omega = -KTN$

$PV = KTN$ $\left(N = \int d\sigma \, e^{-(\varepsilon-\mu)/kT}\right)$

$-N = \int d\sigma \langle n \rangle$

1) Se $\{H, N\} = 0$, $Q = \sum_N \int d\sigma\, e^{N(\mu - H)/kT} =$

$= \sum_N \frac{1}{N!} \left(\int_\Gamma d\sigma\, e^{\mu - \varepsilon/kT} \right)^N = \exp\left(\int d\sigma\, e^{(\mu-\varepsilon)/kT} \right)$

$\Omega = -kT \int d\sigma\, e^{(\mu-\varepsilon)/kT}$; $N = -\frac{\partial \Omega}{\partial \mu} = \int d\sigma \underbrace{e^{\mu-\varepsilon/kT}}_{\langle n \rangle}$

$\Omega = -kTN = PV$

2) Se $\{H, N\} \neq 0$ N minimizza F

$\left.\frac{\partial F}{\partial N}\right|_{i,V} = 0 = \mu$ $\Rightarrow$ Canonico (Gyrem con $\mu = 0$)

3) Interez. necessaria ~~x~~ equilibrio, ma debole x condivise $\Rightarrow$ $PV = NkT$

(~ MORUZZI)

• Teorema H : $S_2 - k \int d\sigma \langle n \rangle \ln \langle n \rangle - 1$

• Derivo $\langle n \rangle$ imponendo ~~$\delta F = 0$~~ S stez. con vincoli U, N

$0 = \delta S' = \delta S + \frac{\delta U}{T} + \frac{\mu}{T} \delta N = \int (\ln \langle n \rangle) \forall \delta n$

$U = \int d\sigma\, \varepsilon \langle n \rangle$

$N = \int d\sigma \langle n \rangle$

$\Rightarrow \langle n \rangle$ $\boxed{\delta S' = 0 = g \langle n \rangle \; \forall n}$

Stat. Mech. quantistiche

WHICH PARTICLE / IN WHICH STATE

- $\mathcal{H}_N = \bigotimes^N \mathcal{H}_1 \Big|_{\text{sym-antisym}}$, $\mathcal{H} = \bigoplus_{N\geq 0}^{\infty} \mathcal{H}_N$

Fisso N, denco $\alpha_1^{n_1} \alpha_2^{n_2} \ldots$ Fisso α, denco n_α

Bose $|\alpha_N\rangle$

- $\mathcal{F}_\alpha$ Bose $|n_\alpha\rangle$, $\mathcal{F} = \bigotimes_{\alpha=1}^{\infty} \mathcal{F}_\alpha$ denco n_α / how many in each?

$\left(\mathcal{F} \to \mathcal{H}\right)$ $\hat{\mathcal{E}}\,|\alpha\rangle = \varepsilon_\alpha |\alpha\rangle$, $\hat{H} = \sum_\alpha \varepsilon_\alpha \hat{N}_\alpha + \text{cost}$

- $\mathrm{Tr}_{\mathcal{H}}(A) = \mathrm{Tr}_{\mathcal{F}}(A) = \prod_\alpha \mathrm{Tr}_{\mathcal{F}_\alpha}(A)$ $\mathcal{F}_\alpha$, n_α / how many in α

- $\hat{N}_\alpha = a_\alpha^\dagger a_\alpha$ o $b_\alpha^\dagger b_\alpha$

$$\left[a_\alpha , a_\beta^\dagger\right] = \left\{ b_\alpha , b_\beta^\dagger \right\} = \delta_{\alpha\beta}$$

$$\left[a_\alpha , a_\beta\right] = \left[a_\alpha^\dagger , a_\beta^\dagger\right] = \left\{ b_\alpha , b_\beta \right\} = \left\{ b_\alpha^\dagger , b_\beta^\dagger \right\} = 0$$

$$|n_\alpha, n_\beta, n_\gamma\rangle = \frac{1}{\sqrt{n_\alpha! n_\beta! n_\gamma!}} \left(a_\alpha^\dagger\right)^{n_\alpha} \left(a_\beta^\dagger\right)^{n_\beta} \left(a_\gamma^\dagger\right)^{n_\gamma} |0\rangle$$

SYM

$$|\alpha, \beta, \gamma\rangle = b_\alpha^\dagger b_\beta^\dagger b_\gamma^\dagger |0\rangle$$

ANTISYM

$$\mathcal{Q}(T,V,\mu) = \mathrm{Tr}_{\mathcal{H}}\left[e^{(\mu N - H)/kT} \right] \qquad [H,N]=0$$

$$\sum_{n_\alpha=0}^{\infty} e^{(\mu-\varepsilon_\alpha) n_\alpha / kT} = \frac{1}{2} + e^{(\mu-\varepsilon_\alpha)/kT}$$

$$\sum_{n_\alpha=0}^{\infty} e^{(\mu-\varepsilon_\alpha) n_\alpha / kT} = \frac{1}{1 - e^{(\mu-\varepsilon_\alpha)/kT}} \qquad \left| \begin{array}{l} e^{\mu-\varepsilon_\alpha/kT} < 1 \\ \mu/kT \\ e < 1 \end{array} \right.$$

$$\mathcal{Q}(T,V,\mu) = \prod_\alpha \left(1 \pm e^{(\mu-\varepsilon_\alpha)/kT} \right)^{\pm 1} \quad \begin{matrix} (F \\ B) \end{matrix}$$

$\mathcal{Q} = \mathrm{Tr}$
$\Omega = -kT \ln \mathcal{Q}$
$N = \frac{\partial \Omega}{\partial \mu}$

$$\Omega = \mp kT \sum_\alpha \ln\left(1 \pm e^{(\mu-\varepsilon_\alpha)/kT} \right)$$

$$N = -\frac{\partial \Omega}{\partial \mu} = \sum_\alpha \frac{1}{e^{(\varepsilon_\alpha - \mu)/kT} \pm 1} = \sum_\alpha \langle n_\alpha \rangle$$

$$\langle n_\alpha \rangle = \frac{1}{e^{(\varepsilon_\alpha - \mu)/kT} \pm 1} \quad \begin{matrix} (F \\ B) \end{matrix} \qquad z = e^{\mu/kT} < 1 \text{ Bosons}$$

Se $[H,N] \neq 0 \Rightarrow \mu = 0$ F

$$\sum_\alpha = \sum_{s} \sum_{n_1} \sum_{n_2} \sum_{n_3} \longrightarrow \frac{V}{h^3} \sum_s \int d^3 p$$

$$\left(p = \frac{\pi \hbar}{L} (n_1, n_2, n_3) \right)$$