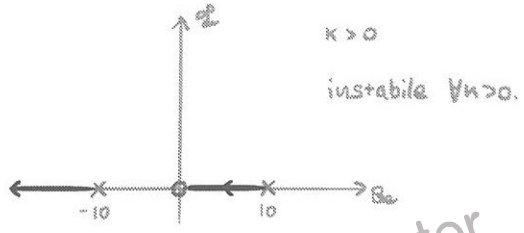
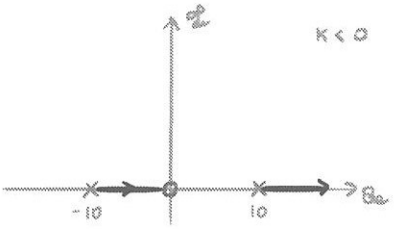
Es1

Domenica 20 febbraio 2022 16:42

Innocenti:
Luogo delle radici di G(s) = (k * s) / ((s + 10) * (s - 10))
E poi trovare un controllore per avere un errore al gradino <= 1% ✓



instabile $\forall K > 0$.



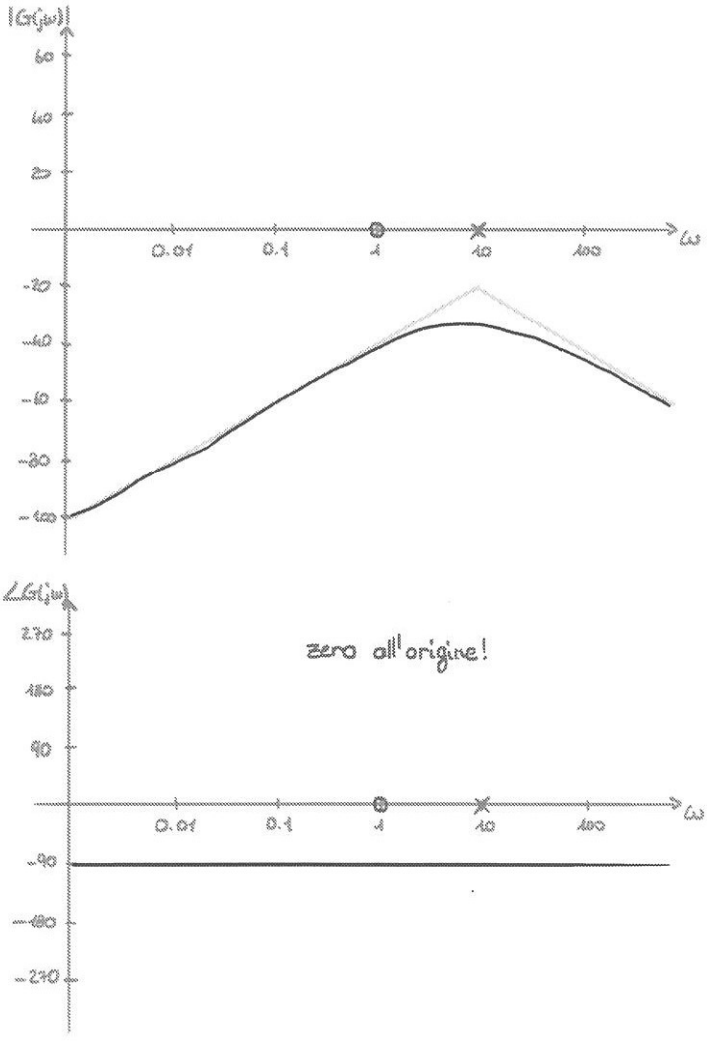
$1 + K G(s) H(s) = 0$

$1 + K \frac{s}{(s+10)(s-10)} = 0$

$(s^2 - 100) + Ks = 0$

↑ C.N. non rispettata

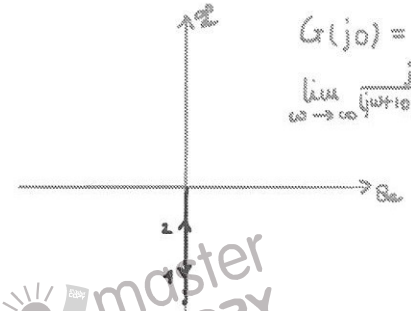
Bode con $K = 2$



$K = \frac{1}{100}$ $\quad 20\log_{10}\frac{1}{100} = -40\,dB$

non si può applicare il criterio di Bode (sistema instabile in OL)

zero all'origine!



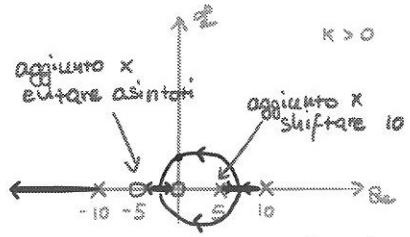
$G(j0) = 0$

$\lim_{\omega \to \infty} \frac{j\omega}{(j\omega+10)(j\omega-10)} = 0$

$G(s) = \frac{s}{(s+10)(s-10)}$ $\quad \varepsilon_p \leq 1\% = 0.01$

- $K(s) = K \longrightarrow$ instabile $\forall K > 0$ (visto prima)

- $K(s) = K$ ⟶ instabile $\forall K > 0$ (visto prima)
- $K(s) = K\frac{s+5}{s-5}$ ← per far shiftare il polo positivo in LHP



punto di distacco/uscita:

$\frac{dK}{ds} = 0$ ove $K = \frac{(s+10)(s-10)(s-5)}{s^2+5s} = \ldots = \begin{cases} s_1 \cong -2.2 \\ s_2 \cong 7 \end{cases}$

$K G(s)H(s) = K\frac{s(s+5)}{(s+10)(s-10)(s+5)}$

K_{cr}: $1 + KG(s)H(s) = 0$

$(s+10)(s-10)(s+5) + Ks(s+5) = 0$

$(s^2-100)(s-5) + Ks^2 + 5Ks = 0$

$s^3 - 5s^2 - 100s + 500 + Ks^2 + 5Ks = 0$

$s^3 + (-5+K)s^2 + (5K-100)s + 500 = 0$

C.N: $\begin{cases} -5+K > 0 \longrightarrow K > 5 \\ 5K-100 > 0 \longrightarrow K > \frac{100}{5} = 20 \end{cases}$

s^3	1	$5K-100$	0
s^2	$-5+K$	500	0
s^1	b_1	0	
s^0	500		

$b_1 = \frac{(K-5)(5K-100) - 500}{K-5}$ $\quad c_1 = \frac{5 c_0 b_1}{b_1}$

$\frac{5K^2 - 100K - 25K + 500 - 500}{K-5} > 0$

$(K > 20)$ $\quad 5K^2 - 125K > 0$

senza lo zero avremmo avuto 2 asintoti in

$x_a = \frac{-10+10+5}{2} = \frac{5}{2}$

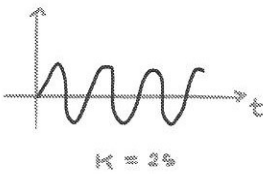
$5K > 125 \longrightarrow K > \frac{125}{5} = 25 \longrightarrow K_{cr} = 25$

sistema asintoticamente stabile per $K > K_{cr} = 25$ $\quad \frac{1}{1+K_p}$ ove $\boxed{K_p = \lim_{s\to 0} |K(s)G(s)|}$

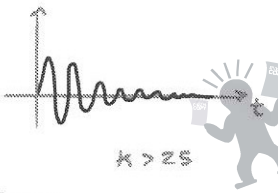
errore al gradino $(K > 25)$: $\varepsilon_r = \frac{1}{1+K} = \frac{1}{1+0} = 1$!!

NON è il guadagno statico!

se si sceglie $K = 25$ la risposta è puramente sinusoidale (poli puramente immaginari)



$K = 25$



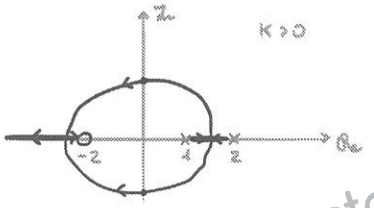
$K > 25$

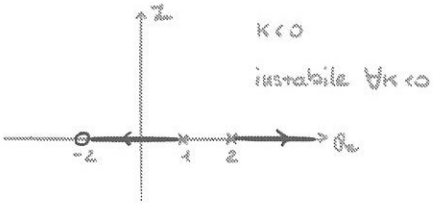
step response

Es2

mercoledì 17 agosto 2022 15:06

Innocenti
- k(s+2)/((s-1)(s-2)) luogo delle radici diretto e inverso, il sistema è stabile, as o instabile?
- Calcolare il k critico della funzione precedente e spiegare la procedura da seguire.
- Con ingresso a gradino a ciclo chiuso quanto vale il minimo errore?
- Cosa serve per applicare il Th. del valore finale?
- Come rendo l'errore al gradino uguale a 0? Come so se un integratore è sufficiente per rendere l'errore uguale a 0?





stabile asintoticamente per $K > K_{cr}$

K_{cr}: $1 + KG(s)H(s) = 0$

$(s-1)(s-2) + K(s+2) = 0$

$s^2 - 2s - s + 2 + Ks + 2K = 0$

$s^2 + (-2-1+K)s + (2+2K) = 0$

C.N. $K - 2 - 1 > 0 \longrightarrow K > 3 = K_{cr}$

s^2	1	$2+2K$
s^1	$-3+K$	0
s^0	b_1	

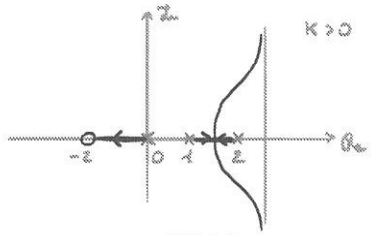
$b_1 = \frac{(-3+K)(2+2K)}{-3+K} = 2+2K > 0 \longrightarrow K > -1$

$\varepsilon_r = \frac{1}{1+K} \longrightarrow 0$ se $K \longrightarrow +\infty$ (in teoria)

TvF: risposta all'impulso del sistema unitaria.

$\varepsilon_r = \lim_{s\to 0} s\varepsilon(s) = \lim_{s\to 0} s S(s) \cancel{\frac{1}{s}} = \lim_{s\to 0} \frac{1}{1+K\frac{K(s+2)}{s(s-1)(s-2)}} = 0$

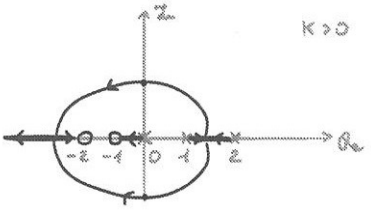
per rendere $\varepsilon_r = 0$ serve $T(s)$ almeno di tipo 1: proviamo $K(s) = \frac{K}{s}$



$X_a = \frac{1+2-(-2)}{3-1} = \frac{5}{2} = 2.5$

instabile $\forall K > 0$

proviamo $K(s) = \frac{K(s+1)}{s}$ (zero in -1)



asintoticamente stabile per $K > K_{cr}$

$KG(s)H(s) = K\frac{(s+1)(s-2)}{s(s-1)(s-2)}$

K_{cr}: $1 + KG(s)H(s) = 0$

$s(s-1)(s-2) + K(s+1)(s+2) = 0$

$s^3 - 2s^2 - s^2 + 2s + Ks^2 + 2Ks + Ks + 2K = 0$

$s^3 + (-3+K)s^2 + (2+3K)s + 2K = 0$

C.N. $\begin{cases} K > 0 \\ 3K > -2 \longrightarrow K > -\frac{2}{3} \\ K > 3 \end{cases}$

s^3	1	$2+3K$	0
s^2	$K-3$	$2K$	0
s^1	b_1	0	
s^0	$2K$		

$b_1 = \frac{(K-3)(2+3K) - 2K}{K-3} = \frac{\cancel{2K} + 3K^2 - 6 - 9K - \cancel{2K}}{K-3} > 0$

$3K^2 - 9K - 6 > 0$

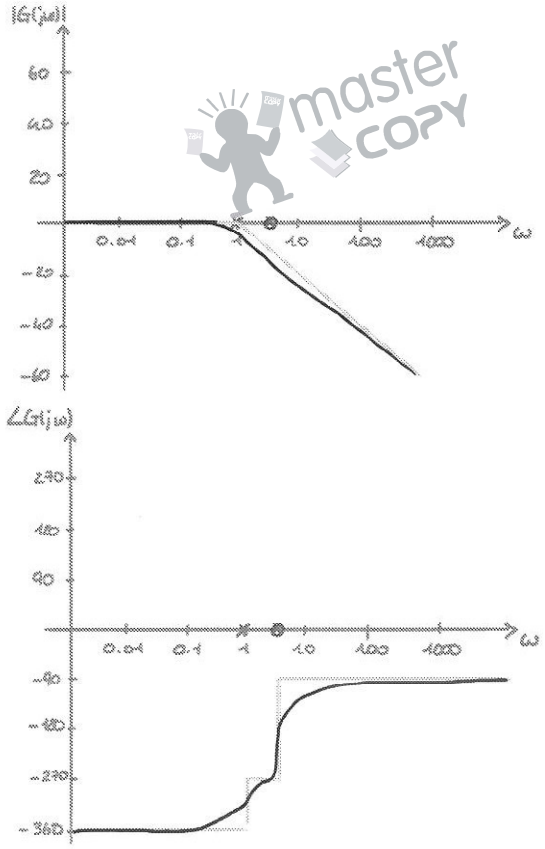
$$K = \frac{9 \pm \sqrt{64 + 4 \cdot 16}}{8} = \begin{cases} K_1 \cong 3.56 \\ K_2 \cong -0.56 \end{cases}$$



$K < -0.56 \ \cup \ K > 3.56$

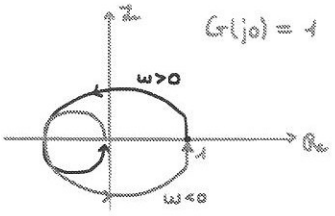
quindi $K_{cr} \cong 3.56$

Bode con $K = 1$ di $G(s) = K \frac{s+2}{(s-1)(s-2)}$ ✓



$K = \frac{2}{2} \longrightarrow 20 \log_{10} |1| = 0$

non si può applicare il criterio di Bode (sistema instabile in CL)

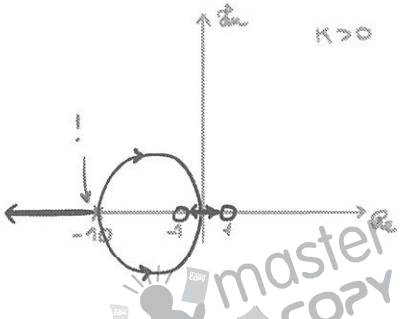


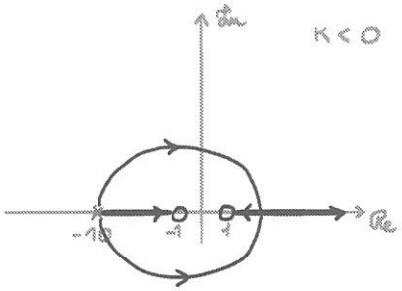
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Es3

venerdì 19 agosto 2022 10.18

- k(s+1)(s-1)/((s+10)^3) luogo delle radici diretto e inverso.
- Cosa si può dire sulla stabilità del sistema?
- Preso k=10 fare i diagrammi di Bode.





asintoticamente stabile per $K < K_{cr}$

K_{cr}: $1 + K G(s) H(s) = 0$

$(s+10)^3 + K(s+1)(s-1) = 0$

$(s+10)(s^2+20s+100) + Ks^2 - K = 0$

$s^3 + 20s^2 + 100s + 10s^2 + 200s + 1000 + Ks^2 - K = 0$

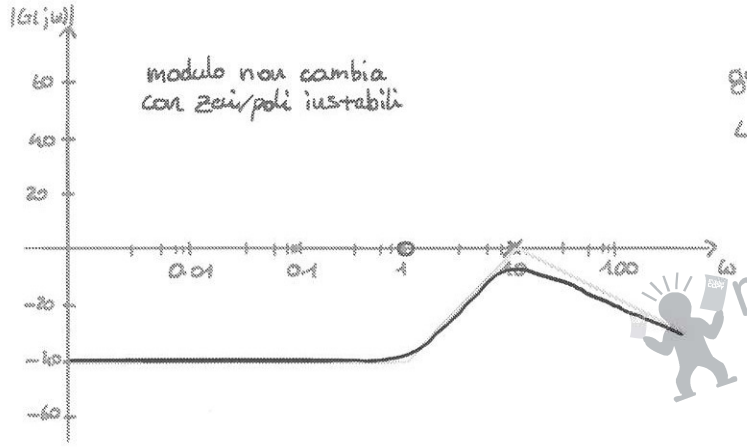
$s^3 + (30+K)s^2 + 300s + (1000-K) = 0$ C.N. $\begin{cases} K > -30 \\ K < 1000 = K_{cr} \end{cases}$

s^3	1	300	0
s^2	30+K	1000-K	0
s^1	b_1	0	
s^0	1000-K		

$b_1 = \frac{(30+K)300 - (1000-K)}{30+K} = \frac{9000 + 300K - 1000 + K}{30+K} > 0$

$30+K > -8000 \longrightarrow K > -\frac{8000}{301} \cong -26.57$

Bode di $G(s) = 10\frac{(s+1)(s-1)}{(s+10)^3}$ $\quad K = -\frac{10}{10^3} = -\frac{1}{10^2}$



modulo non cambia con zeri/poli instabili

guadagno statico: $20\log_{10}\frac{1}{10^2} = -40\,dB$ (·)

$\angle \cdot = 0$

$\downarrow$ 0.01

- stabile in CL
- K > 0
- no

↳ PM $\longrightarrow +\infty$
GM = $40dB|_{\omega\to\infty} > 0$ } stabile in CL

