ALGEBRA LINEARE

$(V)\ (\cdot,\cdot,\cdot)\ (V)$

PRODOTTO CARTESIANO $A \times B = \{(a,b) : a \in A, b \in B\}$

$A_1 \times A_2 \times \dots \times A_n = \{(a_1, a_2, \dots, a_n) : a_i \in A_i\ \forall i = 1 \dots n\}$

$\mathbb{R}^n = \underbrace{\mathbb{R} \times \mathbb{R} \times \dots \times \mathbb{R}}_{n \text{ volte}} = \{(x_1, x_2, \dots, x_n) : x_i \in \mathbb{R}\ \forall i = 1 \dots n\}$

$\mathbb{C}^n = \mathbb{C} \times \mathbb{C} \times \dots \times \mathbb{C} = \{(z_1, z_2, \dots, z_n) : z_i \in \mathbb{C}\ \forall i = 1 \dots n\}$

$a = (a_1, \dots, a_n)$ COMPONENTI — $b = (b_1, \dots, b_n)$ — $\lambda, \mu \in \mathbb{R}$

VETTORI — SCALARI

- $a + b = (a_1 + b_1, a_2 + b_2, \dots, a_n + b_n)$

$a = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ $b = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ → non si può derivare la somma

$a = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$ $b = \begin{pmatrix} 2 \\ 0 \\ 1 \end{pmatrix}$ $a + b = \begin{pmatrix} 3+2 \\ 2+0 \\ 1+1 \end{pmatrix} = \begin{pmatrix} 5 \\ 2 \\ 2 \end{pmatrix}$

- $\lambda a = a\lambda = (\lambda a_1, \lambda a_2, \dots, \lambda a_n)$ SCALARE × VETTORE = VETTORE

$3\begin{pmatrix} 1 \\ 2 \\ 1 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \\ 3 \end{pmatrix}$

$0 = (0, 0, 0, \dots, 0)$ $\quad a + 0 = (a_1 + 0, a_2 + 0, \dots, a_n + 0) = (a_1, a_2, \dots, a_n)$

$-a = (-1)\,a = (-a_1, -a_2, \dots, -a_n)$

$a + (-a) = (a_1 - a_1, a_2 - a_2, \dots, a_n - a_n) = (0, 0, 0, \dots, 0) = 0$

$0a = 0$ (0 scalare, 0 vettore) $\qquad \lambda 0 = 0$ (0 vettore)

② $a + b = b + a$ COMMUTATIVA (ABELIANA)

① $a + (b + c) = (a + b) + c$ ASSOCIATIVA

$a + b + c$

③ $\exists 0 : a + 0 = 0 + a = a$ $\quad \exists -a : a + (-a) = 0$ $\quad \forall a \in \mathbb{R}^n$

$2x = 3x - 3 \qquad 2x - 3x = 3x - 3 - 3x$

③ $\lambda(a + b) = \lambda a + \lambda b$ $\quad \forall \lambda \in \mathbb{R},\ \forall a, b \in \mathbb{R}^n$ DISTRIBUTIVA

④ $(\lambda + \mu)a = \lambda a + \mu a$ $\quad \forall \lambda, \mu \in \mathbb{R},\ \forall a \in \mathbb{R}^n$

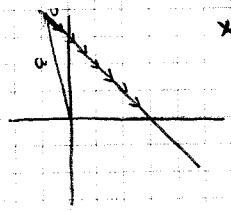
⑤ $\lambda(\mu a) = (\lambda\mu)a$ $\quad \forall \lambda, \mu \in \mathbb{R}\ \forall a \in \mathbb{R}^n$ (vettore)

⑥ $1 \cdot a = a$ $\quad \forall a \in \mathbb{R}^n$

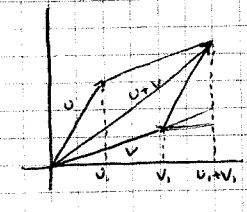
$\lambda a = 0 \iff \lambda = 0$ oppure $a = 0$ ANNULLAMENTO DEL PRODOTTO

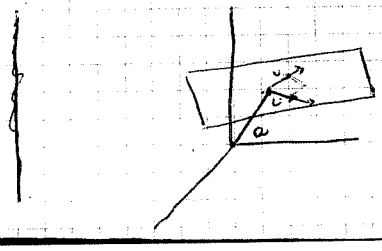
VETTORI → spostamenti a partire dall'origine

→ una vale l'annullamento



$x(t) = a + bv$ $\quad \begin{cases} x_1(t) = a_1 + tv_1 \\ x_2(t) = a_2 + tv_2 \end{cases}$





$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix} + \lambda \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix} + \mu \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$

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$x=\begin{pmatrix}x_1\\ \vdots\\ x_n\end{pmatrix}$ $ax=\begin{pmatrix}ax_1\\ \vdots\\ ax_n\end{pmatrix}$ $ax=0=\begin{pmatrix}0\\ \vdots\\ 0\end{pmatrix}$ $\begin{pmatrix}ax_1\\ \vdots\\ ax_n\end{pmatrix}=\begin{pmatrix}0\\ \vdots\\ 0\end{pmatrix}$ $\begin{cases}ax_1=0\\ \vdots\\ ax_n=0\end{cases}$ $a=0$ oppure $a\neq 0$ $\begin{cases}x_1=0\\ \vdots\\ x_n=0\end{cases}$ oppure

SISTEMI LINEARI (SISTEMI DI EQ DI I GRADO IN PIÙ VARIABILI)

$a_1x_1+a_2x_2+\dots+a_nx_n=b$

$\begin{cases}a_{11}x_1+a_{12}x_2+\dots+a_{1n}x_n=b_1\\ \vdots\\ a_{m1}x_1+a_{m2}x_2+\dots+a_{mn}x_n=b_m\end{cases}$ m EQUAZIONI, n INCOGNITE

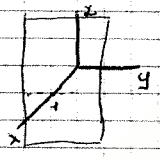
$\begin{pmatrix}a_{11}\\ a_{21}\\ \vdots\\ a_{m1}\end{pmatrix}$ A_1 $\in\mathbb{R}^m$ $\begin{pmatrix}a_{12}\\ \vdots\\ a_{m2}\end{pmatrix}$ A_2 $\in\mathbb{R}^m$ $\begin{pmatrix}a_{1n}\\ \vdots\\ a_{mn}\end{pmatrix}$ A_n $\in\mathbb{R}^m$ $\begin{pmatrix}b_1\\ \vdots\\ b_m\end{pmatrix}$ B $\in\mathbb{R}^m$

$x_1\begin{pmatrix}a_{11}\\ a_{21}\\ \vdots\\ a_{m1}\end{pmatrix}+x_2\begin{pmatrix}a_{12}\\ \vdots\\ a_{m2}\end{pmatrix}+\dots+x_n\begin{pmatrix}a_{1n}\\ \vdots\\ a_{mn}\end{pmatrix}=\begin{pmatrix}b_1\\ \vdots\\ b_m\end{pmatrix}$ $x_1A_1+x_2A_2+\dots+x_nA_n=B$ $\mathbb{R}^m$

$0x=1\to$ NO SOLUZIONI! $0x_1+0x_2+\dots+0x_n=b\neq 0$ ~~IMPOSSIBILE~~ NO SOLUZIONI → INSOLUBILE!

$\exists(x,y)\in\mathbb{R}^2: x=1$ $1x+0y=1$

$\exists(x,y,z)\in\mathbb{R}^3: x=1$ $(1,y,z)$



$0x_1+0x_2+\dots+0x_n=0$

$x+y+z=1$ $x=1-y-z$ sol $\begin{pmatrix}1-\alpha-\beta\\ \alpha\\ \beta\end{pmatrix}$ $\alpha,\beta\in\mathbb{R}$ $\begin{pmatrix}1\\ 0\\ 0\end{pmatrix}+\begin{pmatrix}-1\\ 1\\ 0\end{pmatrix}\alpha+\begin{pmatrix}-1\\ 0\\ 1\end{pmatrix}\beta$

ALGORITMO DI ELIMINAZIONE (DI RIDUZIONE A SCALA)-GAUSS

$\begin{cases}2x+3y+z=1\\ \to y+z=3\\ \to 4z=7\end{cases}$ SISTEMA TRIANGOLARE $z=7/4$

2	3	1	1
0	1	2	3
0	0	4	7

$\begin{cases}x_2+2x_3+x_4+3x_5+x_6=1\\ 3x_4+2x_5-x_6=3\\ +3x_5+2x_6=9\end{cases}$ SISTEMA SCALA

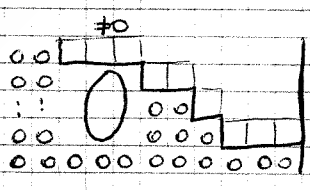
$3x_5=9-2x_6$ FISSATE x_6 e x_3 si ricavano tutte le altre incognite

SOSTITUZIONE ALL'INDIETRO

x_1	x_2	x_3	x_4	x_5	x_6	
0	1	2	1	3	1	1
0	0	0	3	2	-1	3
0	0	0	0	3	2	9

PIVOT → 1 per colonne

NON PIVOT → si sceglie a caso



$\begin{cases}x_2-2x_3+x_4+x_6=1\\ x_3-2x_4+x_5+x_6=-1\\ x_5-x_6=2\end{cases}$ $x_5=2+x_6$ $x_3=-1+2x_4-x_5-x_6$

x_1	x_2	x_3	x_4	x_5	x_6	
0	1	-2	1	0	1	1
0	0	1	-2	1	1	-1
0	0	0	0	1	-1	2

$\begin{pmatrix}x_1\\ x_2(x_4,x_6)\\ x_3(x_4,x_6)\\ x_4\\ x_5(x_6)\\ x_6\end{pmatrix}$

x_1 ... x_n	
a_{11} ... a_{1n}	b_1
a_{21} ... a_{2n}	b_2
⋮	
a_{m1} ... a_{mn}	b_m

① Scambiare righe senza alterare la soluzione

② Scambiare colonne scambiando di posto alle incognite

$x_1 \dots x_n$ $\begin{cases} a_1x_1 + a_2x_2 + \dots + a_nx_n = A \\ b_1x_1 + b_2x_2 + \dots + b_nx_n = B \end{cases}$

$\begin{cases} a=b \\ c=d \end{cases} \Rightarrow \begin{cases} a+c=b+d \\ a=b \end{cases}$

x_1	x_2	x_3	
1	2	3	1
2	0	1	1
1	3	2	0

$II-2I$, $III-I$ $\Longleftrightarrow$ $II-III$ $\Longleftrightarrow$ $III+4II$

$\begin{cases} x+2y+3z=1 \\ y-z=-1 \\ -9z=-5 \end{cases}$ $\begin{cases} x=2/9 \\ y=-4/9 \\ z=5/9 \end{cases}$

1	0	1	1
2	1	3	1
3	2	5	2

$II-2I$, $III-3I$ $\Longleftrightarrow$ $III-2II$ $\Longleftrightarrow$

1	0	1	1
0	1	1	−1
0	0	0	1

INCOMPATIBILE!

x_1	x_2	x_3	
0	1	1	0
1	2	1	0
2	0	1	0
3	2	3	0

SISTEMA OMOGENEO

$II-2I \to IV$, $III-2I \to V$, $IV-3I \to VI$, $V-VI$, $II \leftrightarrow III$, $V-3II \to VI$

x_1	x_3	x_2	
1	1	2	0
0	1	1	0
0	−1	−1	0
0	0	−4	0
0	0	−3	0
0	0	0	

$III+II$, $IV-\frac{3}{4}III$

$\begin{cases} x_1+x_3+2x_2=0 \\ x_3+x_2=0 \\ -4x_2=0 \end{cases}$

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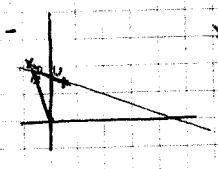
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x_1	x_2	x_3	
1	3	4	−1
0	2	2	−2
2	1	3	4

$II-2I \to IV$, $III+2/5\,IV \to V$

1	3	4	−1
2	−1	3	4
0	−2	−2	−2
0	−5	−5	6
0	0	0	2/5

INCOMPATIBILE



$x(t) = x_0 + tv$ $\quad y(s) = y_0 + sv$ RETTE PARALLELE CON VETTORI VETTORI MULTIPLI

$x_0 + tv = y_0 + sv$ COINCIDENTI $\quad x_0 + tv = y_0 + sv$ INTERSECANTI

$\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} + t\begin{pmatrix} 1 \\ 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix} + s\begin{pmatrix} 1 \\ -1 \\ 1 \end{pmatrix}$ $\quad \begin{cases} 1+t=2+s \\ 0+t=1+s \\ 1+0=1+s \end{cases}$ $\quad \begin{cases} t-s=1 \\ -t-s=1 \\ s=0 \end{cases}$ $\quad s=0$, $t=1$ RETTE INCIDENTI (1 SOL)

$\begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} 2 \\ 1 \\ -1 \end{pmatrix} = \begin{pmatrix} 3 \\ 0 \\ -1 \end{pmatrix} + s\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}$ $\quad \begin{cases} 1+2t=3+s \\ 0+t=0+s \\ 0+t=-1+s \end{cases}$ $\quad \begin{cases} -s+2t=2 \\ -s+t=0 \\ -s+t=-1 \end{cases}$

−1	2	2
−1	1	0
−1	1	−1
0	−1	−2
0	−1	−1
0	0	−1

RETTE SGHEMBE (ϕ SOL, vett. multipli)

$\begin{pmatrix} 1 \\ 0 \end{pmatrix} + t\begin{pmatrix} -1 \\ -1 \end{pmatrix} = \begin{pmatrix} 0 \\ 1 \end{pmatrix} + s\begin{pmatrix} 2 \\ -2 \end{pmatrix}$ $\quad \begin{cases} 1-t=2s \\ t=-2s+1 \end{cases}$ $\quad \begin{cases} t+2s=1 \\ t+2s=-1 \end{cases}$ RETTE COINCIDENTI (∞ SOL)

$\begin{cases} x+y-2z=1 \\ 2x-y-z=3 \end{cases}$

1	−1	1
2	−1	3
0	−3	1

$\begin{cases} x+y-2z=1 \\ -3y+3z=1 \end{cases}$ $\quad y = \frac{3z-1}{3}$ $\quad x = 1-y+2z = 1+\frac{1}{3}-\frac{3z}{3}+2z = \frac{4}{3}+z$

ALGORITMO DI GAUSS-JORDAN

1	2	3	1
0	2	5	0
0	0	2	2

1	0	0	1
0	2	0	2
0	0	3	3

$\begin{cases} x_1=1 \\ 2x_2=2 \\ 3x_3=3 \end{cases}$ SISTEMA DIAGONALE (DISACCOPPIATO)

1	0	0	1
0	1	0	3
0	0	1	5

MATRICE UNITARIA

1	2	3	1
0	2	5	0
0	0	2	2
0	0	1	1

$\frac{1}{2}III \Longleftrightarrow II-5III$

1	2	3	1
0	2	0	−5
0	0	1	1

$\frac{1}{2}II$, $\frac{1}{3}III$

1	2	3	1
0	1	0	−5/2
0	0	1	1

$I-(2II+3III)$ $\Longrightarrow$

1	0	0	3
0	1	0	−5/2
0	0	1	1

soluzione

$$\left(\begin{array}{ccc|c} 1&0&1&1\\ 2&1&3&2\\ 1&1&1&2 \end{array}\right) \quad \left(\begin{array}{ccc|c} 1&0&1&3\\ 2&1&3&2\\ 1&1&1&4 \end{array}\right) \Rightarrow \left(\begin{array}{ccc|c} 1&0&1&1&3\\ 2&1&3&2&2\\ 1&1&1&2&4 \end{array}\right)$$

$$\left(\begin{array}{ccc|cc} 1&0&0&\square&\square\\ 0&1&0&&\\ 0&0&1&&\\ \end{array}\right)$$

A posso risolvere contemporaneamente più problemi che differiscono solo per i termini noti

set I set II

$$\left(\begin{array}{ccc|cc} 1&0&1&1&3\\ 2&1&3&2&2\\ 1&1&1&2&4\\ 0&1&1&0&4\\ 0&1&0&1&1\\ 0&1&1&0&4\\ 0&0&1&-1&-5\\ 1&0&0&2&8 \end{array}\right) \quad \begin{array}{l} \text{I}-\text{III}\to\text{VIII}\\ \text{II}-2\text{I}\to\text{IV}\\ \text{III}-\text{I}\to\text{V}\\ \text{IV}\to\text{VI}\\ \\ \text{VII}-\text{V}\to\text{VIII} \end{array} \qquad \left(\begin{array}{ccc|cc} 1&0&0&2&8\\ 0&1&0&1&1\\ 0&0&1&-1&-5 \end{array}\right)$$

$$\begin{pmatrix} x\\ y\\ z \end{pmatrix} = \alpha v + \beta w = \alpha \begin{pmatrix} v_1\\ v_2\\ v_3 \end{pmatrix} + \beta \begin{pmatrix} w_1\\ w_2\\ w_3 \end{pmatrix} \qquad \begin{pmatrix} x\\ y\\ z \end{pmatrix} = \gamma \begin{pmatrix} v_1'\\ v_2'\\ v_3' \end{pmatrix} + \delta \begin{pmatrix} w_1'\\ w_2'\\ w_3' \end{pmatrix}$$

$$\begin{cases} x = 2\alpha + 3\beta\\ y = 3\alpha + 2\beta\\ z = \alpha + \beta \end{cases} \qquad \begin{cases} x = \alpha - \beta\\ y = 2\alpha + 3\beta\\ z = -2\alpha + \beta \end{cases} \qquad \begin{array}{l} \alpha_1 A_1 + \alpha_2 A_2 \quad \pi\\ \beta_1 B_1 + \beta_2 B_2 \quad \rho \end{array} \qquad \begin{array}{l} A_1, A_2, B_1, B_2 \in \mathbb{R}^3\\ \alpha_1, \alpha_2, \beta_1, \beta_2 \in \mathbb{R} \end{array}$$

$$\begin{pmatrix} x\\ y\\ z \end{pmatrix} \in G\{\alpha_1 A_1 + \alpha_2 A_2\} \iff \exists \alpha_1, \alpha_2 : \begin{pmatrix} x\\ y\\ z \end{pmatrix} = \alpha_1 A_1 + \alpha_2 A_2 \qquad \forall \beta_1, \beta_2 \ \exists \alpha_1, \alpha_2 : \alpha_1 A_1 + \alpha_2 A_2 = \underbrace{\beta_1 B_1 + \beta_2 B_2}_{\text{generico punto di } \rho \text{ al variare di } \beta_1, \beta_2 \in \mathbb{R}}$$

$$\bar{\alpha}_1 A_1 + \bar{\alpha}_2 A_2 = B_1 \qquad \bar{\bar{\alpha}}_1 A_1 + \bar{\bar{\alpha}}_2 A_2 = B_2$$

$\Downarrow$ LINEARITÀ DEI SISTEMI LINEARI $\Downarrow$ i due piani coincidono

$$(\bar{\alpha}_1 \beta_1) A_1 + (\bar{\alpha}_2 \beta_1) A_2 = \beta_1 B_1 \qquad (\bar{\bar{\alpha}}_1 \beta_2) A_1 + (\bar{\bar{\alpha}}_2 \beta_2) A_2 = \beta_2 B_2$$

$$(\bar{\alpha}_1 \beta_1 + \bar{\bar{\alpha}}_1 \beta_2) A_1 + (\bar{\alpha}_2 \beta_1 + \bar{\bar{\alpha}}_2 \beta_2) A_2 = \beta_1 B_1 + \beta_2 B_2$$

$$\gamma_1 A_1 + \gamma_2 A_2 \mid B_1 \ B_2$$

NORMA-MODULO-LUNGHEZZA $|a| = \left(\sum_{i=1}^{n} a_i^2\right)^{1/2} \quad a \in \mathbb{R}^n$

① $|a| \geq 0$

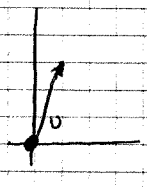
② $|a| = 0 \iff a = 0$

③ $|\lambda a| = |\lambda||a|$ OMOGENEITÀ $\quad |\lambda a| = \sqrt{\sum (\lambda a_i)^2} = \sqrt{\lambda^2 \sum a_i^2} = |\lambda||a|$

④ $|x+y| \leq |x| + |y|$ DISUGUAGLIANZA TRIANGOLARE

VERSORI $u \neq 0 \quad \hat{u} = \frac{1}{|u|} \cdot u \qquad |\hat{u}| = \left|\frac{1}{|u|} u\right| = \left|\frac{1}{|u|}\right| |u| = \frac{1}{|u|} |u| = 1$

$v = (1,1,1) \quad |v| = \sqrt{3} \quad \hat{v} = (1/\sqrt{3}, 1/\sqrt{3}, 1/\sqrt{3})$



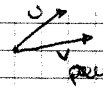
$|F| = G\frac{m_1 m_2}{d^2} \qquad |F| = \frac{G m_1 m_2}{|u|^2} \qquad \hat{u} = \frac{u}{|u|} \qquad F = \frac{G m_1 m_2}{|u|^2}\left(-\frac{u}{|u|}\right) = -\frac{G m_1 m_2}{|u|^3} \cdot u$

la base è diretta dal corpo al polo

$m\ddot{u} = F(u)$

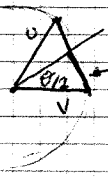
$\ddot{u}$: accelerazione

PRODOTTO SCALARE $u \cdot v = |u||v| \cos \widehat{uv} = \sum_{i=1}^{n} u_i \cdot v_i$ (angolo tra u e v)



coniugo i versori e prendo la bisettrice

prendo altro positivo verso $\quad |u| = |v| = 1$



$(u-v)_i = u_i - v_i \qquad \cos 2\alpha = 1 - 2\sin^2\alpha$

$\frac{1}{2}|u-v| = 1 \cdot \sin\theta/2 \qquad \sin\theta/2 = \frac{1}{2}|u-v| \qquad 4\sin^2\frac{\theta}{2} = |u-v|^2$

$$1 - 2\sin^2\frac{\theta}{2} = 1 - \frac{1}{2}|u-v|^2 = 1 - \frac{1}{2}\sum_{i=1}^{n}(u_i - v_i)^2 = 1 - \frac{1}{2}\sum_{i=1}^{n}(u_i^2 + v_i^2 - 2u_i v_i) =$$

$$= 1 - \frac{1}{2}\left(\sum_{i=1}^{n} u_i^2 + \sum_{i=1}^{n} v_i^2 - 2\sum_{i=1}^{n} u_i v_i\right) = 1 - \frac{1}{2}\left[1 + 1 - 2\sum_{i=1}^{n} u_i v_i\right] = \sum_{i=1}^{n} u_i v_i \Rightarrow \text{l'angolo non dipende dal modulo} \Rightarrow \text{si prende il versore}$$

(1 = 1 per i versori)

$x_1 = 1 - 3x_3$

$y_2 = 3 - 5x_3 = -1 + 3x_3 + 2 - 2x_3$

INCOMPATIBILE

$x_1 = 2$, $x_2 = 0$, $x_3 = 1$

$u, v \neq 0 \quad \hat{u} = \left(\frac{u_1}{|u|}, \frac{u_2}{|u|}, \dots, \frac{u_n}{|u|}\right) \quad \cos\widehat{uv} = \cos\widehat{\hat{u}\hat{v}} = \sum_{i=1}^{n} \frac{u_i}{|u|} \cdot \frac{v_i}{|v|} = \frac{1}{|u||v|} \sum_{i=1}^{n} u_i \cdot v_i$

$\forall u, v \in \mathbb{R}^n \quad uv = \sum_{i=1}^{n} u_i \cdot v_i \qquad u, v \neq 0 \Rightarrow \cos\widehat{uv} = \frac{uv}{|u||v|}$

- $u = \begin{pmatrix} 1 \\ 2 \\ -3 \end{pmatrix} \quad |u| = \sqrt{1^2 + 2^2 + (-3)^2} = \sqrt{14} \quad \hat{u} = \left(1/\sqrt{14}, 2/\sqrt{14}, -3/\sqrt{14}\right)$

- $u(1,2,-1) \cdot v(-2,-1,0) = 1(-2) + 2(-1) + (-1)\cdot 0 = -2 + (-2) = -4$

$|u| = |(1,2,-1)| = \sqrt{6} \quad |v| = |(-2,-1,0)| = \sqrt{5} \quad \cos\widehat{uv} = -4/\sqrt{30}$

u, v ORTOGONALI $\Rightarrow \cos\widehat{uv} = 0 \Rightarrow u \cdot v = 0$

PROPRIETÀ DEL PRODOTTO SCALARE

① $u \cdot u = \sum_{i=1}^{n} u_i \cdot u_i = |u|^2 \geq 0$

② $u \cdot u = \sum_{i=1}^{n} u_i \cdot u_i = |u|^2 = 0 \Leftrightarrow u = 0$

③ $u \cdot v = v \cdot u \quad \sum_{i=1}^{n} u_i \cdot v_i = \sum_{i=1}^{n} v_i \cdot u_i$

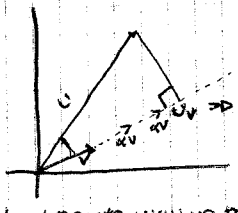
④ $(\alpha u + \beta v) w = \alpha(uw) + \beta(vw)$

- $(1,0,0) \cdot (0,1,0) \cdot (0,0,1) = 0$ $\quad [(1,0,0) \times (1,0,0)] \cdot (0,1,0) = 1 \cdot (0,1,0) = (0,1,0)$

(prodotto scalare; scalare × vettore)

$(1,0,0) \cdot [(1,0,0) \cdot (0,1,0)] = (1,0,0) \cdot 0 = (0,0,0) \Rightarrow$ NON VALE LA PROPRIETÀ ASSOCIATIVA!

PRINCIPIO DI PROIEZIONE



$P_v(u)$

la distanza minima tra αv e u è la proiezione ortogonale

$v \neq 0 \quad \frac{v}{|v|} \quad |u| \cos\widehat{uv} \cdot \frac{v}{|v|} = |u| \frac{uv}{|u||v|} \cdot \frac{v}{|v|} \quad u_v = \frac{uv}{|v|^2} \cdot v$ PROIEZIONE DI u SU v

PROPRIETÀ: ① $u = \alpha v \quad u_v = \frac{uv}{|v|^2} \cdot v \Rightarrow u_v = \frac{(\alpha v)v}{|v|^2} v = \frac{\alpha(vv)}{|v|^2} v = \alpha v$

② $(u - u_v)v = 0$ (cioè le proiezioni sono perpendicolari)

$uv - u_v \cdot v = uv - \left(\frac{uv}{|v|^2} \cdot v\right) \cdot v = uv - \frac{uv}{|v|^2}(v \cdot v) = 0$

③ $|u+v|^2 = |u|^2 + |v|^2$ (se vettori u e v sono perpendicolari)

$(u+v)\underbrace{(u+v)} = uw + vw = u(u+v) + v(u+v) = u^2 + uv + vu + v^2 = |u|^2 + \underbrace{2uv}_{=0} + |v|^2$

Proprietà

- $|u - \alpha v|^2 = |\underbrace{u - u_v}_{\perp v} + \underbrace{u_v - \alpha v}_{\text{multiplo di } v}|^2 = |u - u_v|^2 + |u_v - \alpha v|^2 \geq |u - u_v|^2$

u $\quad v$

$|u_v| \leq |u| \qquad |u|^2 = |\underbrace{u - u_v}_{\perp v} + \underbrace{u_v}_{\text{multiplo di } v}|^2 \overset{\text{Pitag}}{=} |u - u_v|^2 + |u_v|^2 \geq |u_v|^2$ (la proiezione è minore o uguale del vettore da cui è proiettato)

u $\quad 0$