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$$I = \frac{dQ}{dt}$$

$$L_{AB} = \int_A^B \vec{F} \cdot d\vec{r} = U_A - U_B$$

$$V = \frac{U}{Q}$$

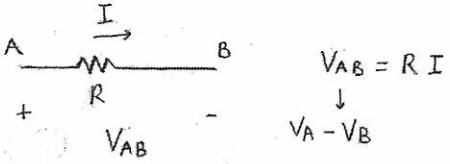
$$\vec{E} = \frac{\vec{F}}{Q}$$

$$\int_A^B \vec{E} \cdot d\vec{r} = V_A - V_B$$

$$1.6 \times 10^{-19}\ \text{J} = 1\ \text{eV}$$



- Resistore



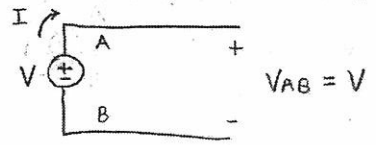
$V_{AB} = R\,I$ ↓ $V_A - V_B$

$P_{diss} = R\,I^2$

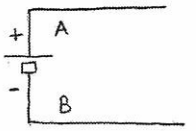
$U_A - U_B = E_{diss}$

- Generatori non controllati

di tensione ideale

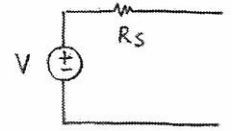


$V_{AB} = V$

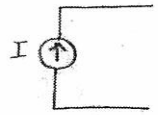
(

se continuo)

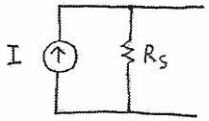
di tensione reale



di corrente ideale

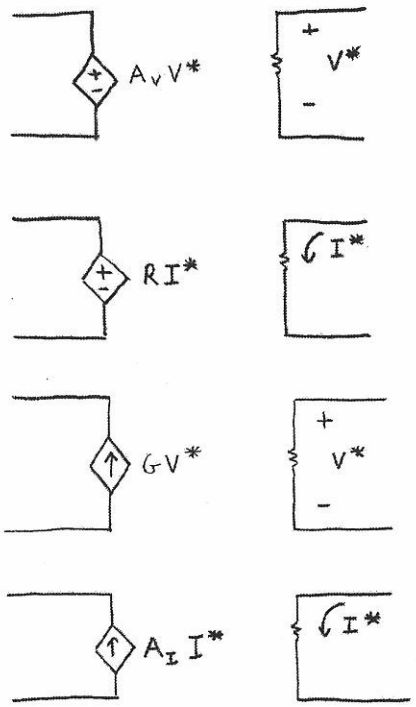


di corrente reale

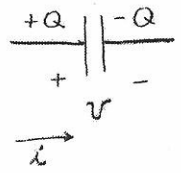




• Generatori controllati



• Capacitore



$$C = \frac{Q}{v} \rightarrow Q = Cv$$

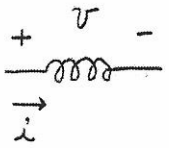
$$v = \frac{Q}{C} = \frac{1}{C}\int i\,dt$$

$$i = \frac{dQ}{dt} = C\frac{dv}{dt}$$

$$E_e = \frac{1}{2} C v^2$$



• Induttore



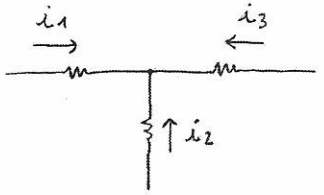
$$\phi = L i$$

$$v = \frac{d\phi}{dt} = L\frac{di}{dt}$$

$$\frac{di}{dt} = \frac{v}{L} \rightarrow i = \frac{1}{L}\int v\,dt$$

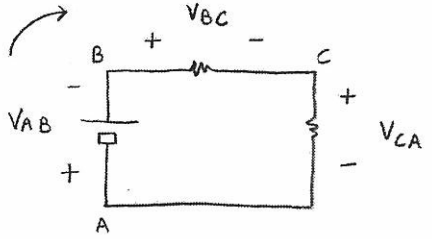
$$E_m = \frac{1}{2} L i^2$$

• Primo principio di Kirchhoff



$$i_1 + i_2 + i_3 = 0$$

• Secondo principio di Kirchhoff



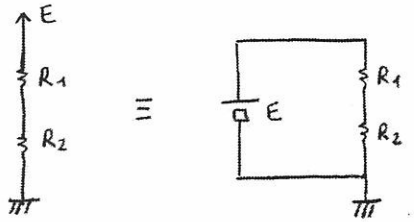
$$V_{AB} + V_{BC} + V_{CA} = 0$$

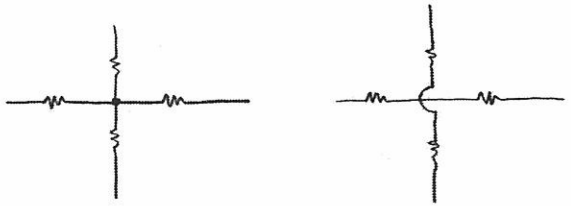
(infatti $V_A - V_B + V_B - V_C + V_C - V_A = 0$)

• Riguardo alla simbologia:

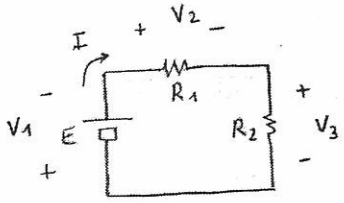


nodo di massa





• Esempio



(R_1 e R_2 in serie)

$$V_1 + V_2 + V_3 = 0$$

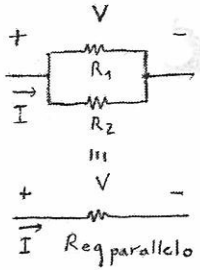
$$-E + R_1 I + R_2 I = 0$$

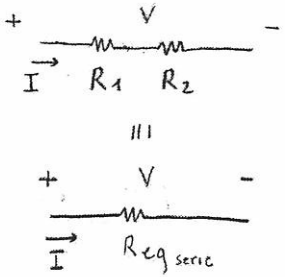
$$E = R_1 I + R_2 I = (R_1 + R_2) I$$

$$I = \frac{E}{R_1 + R_2}$$

$$V_3 = R_2 I = E \frac{R_2}{R_1 + R_2}$$

partitore di tensione





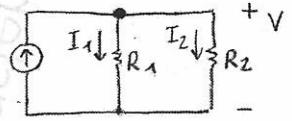
$$V = (R_1 + R_2) I$$

$$\frac{V}{I} = R_1 + R_2 = R_{eq\ serie}$$

in generale:

$$R_{eq\ serie} = \sum_i R_i$$

• Altro esempio



$$I - I_1 - I_2 = 0$$

$$I = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2} =$$

$$= V \left(\frac{1}{R_1} + \frac{1}{R_2} \right) \rightarrow$$

$$\frac{V}{I} = \frac{1}{\frac{1}{R_1} + \frac{1}{R_2}} = R_{eq\ parallelo} = \frac{1}{\frac{R_2 + R_1}{R_1 R_2}} = \frac{R_1 R_2}{R_1 + R_2}$$

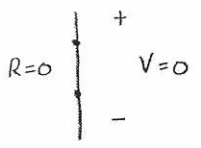
$$R_{eq\ parallelo} = R_1 // R_2$$

in generale: $R_{eq\ parallelo} = \dfrac{1}{\sum_i \frac{1}{R_i}}$

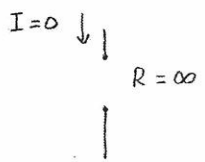
$$V = I\ (R_1 // R_2) = I \frac{R_1 R_2}{R_1 + R_2}$$

$$I_2 = \frac{V}{R_2} = I \frac{R_1 \cancel{R_2}}{R_1 + R_2} \frac{1}{\cancel{R_2}} = I \frac{R_1}{R_1 + R_2}$$

partitore di corrente



cortocircuito



ramo aperto

- Esistono vari metodi per risolvere i circuiti utilizzando i principi di Kirchhoff:
  - correnti di ramo
  - correnti di maglia (o di corda)
  - insiemi di taglio (e, come caso particolare, tensioni di nodo)

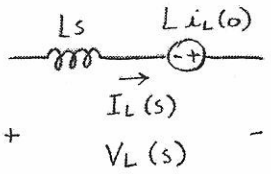
- Dominio di Laplace

$$\mathcal{L}\left[\frac{d}{dt} f(t)\right] = s \cdot \mathcal{L}\left[f(t)\right] - f(0)$$

$$\mathcal{L}\left[\int_{-\infty}^{t} f(\tau)\, d\tau\right] = \frac{\mathcal{L}\left[f(t)\right]}{s} + \frac{\int_{-\infty}^{0} f(\tau)\, d\tau}{s}$$

$$v_L(t) = L \frac{d i_L(t)}{dt} \rightarrow$$

$$V_L(s) = L\left[s I_L(s) - i_L(0)\right] = L s I_L(s) - L i_L(0)$$

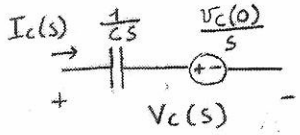


$$Z_L = Ls$$

$$v_C(t) = \frac{1}{C}\int_{-\infty}^{t} i_C(\tau)\, d\tau \rightarrow$$

$$V_C(s) = \frac{1}{C}\left[\frac{I_C(s)}{s} + \frac{\int_{-\infty}^{0} i_C(\tau)\, d\tau}{s}\right] =$$

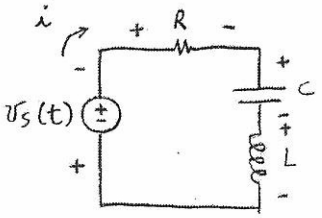
$$= \frac{I_C(s)}{Cs} + \frac{\frac{1}{C}\int_{-\infty}^{0} i_C(\tau)\, d\tau}{s} = \frac{I_C(s)}{Cs} + \frac{v_C(0)}{s}$$



$$Z_C = \frac{1}{Cs}$$



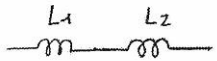
$$v_R(t) = R\, i_R(t) \rightarrow V_R(s) = R\, I_R(s) \rightarrow Z_R = R$$



$$-v_S(t) + R i + \frac{1}{C}\int_{-\infty}^{t} i(\tau)\, d\tau + L\frac{di}{dt} = 0$$

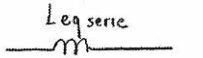
$$v_S(t) = R i + \frac{1}{C}\int_{-\infty}^{t} i(\tau)\, d\tau + L\frac{di}{dt}$$

$$V_S(s) = RI + \frac{1}{Cs} I + LsI$$

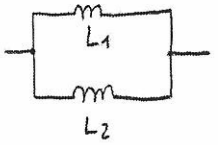


$$Z_{eq\ serie} = Z_1 + Z_2 = sL_1 + sL_2 =$$

$$= s(L_1 + L_2) = s\, L_{eq\ serie}$$

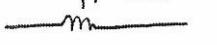


$$L_{eq\ serie} = L_1 + L_2$$



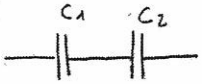
$$Z_{eq\ parallelo} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2}} = \frac{1}{\frac{1}{sL_1} + \frac{1}{sL_2}} =$$

$$= \frac{1}{\frac{1}{s}\left(\frac{1}{L_1} + \frac{1}{L_2}\right)} = s\,\frac{1}{\frac{1}{L_1} + \frac{1}{L_2}} =$$



$$= s\, L_{eq\ parallelo}$$

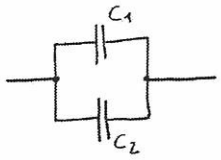
$$L_{eq\ parallelo} = \frac{1}{\frac{1}{L_1} + \frac{1}{L_2}}$$



$$Z_{eq\,serie} = Z_1 + Z_2 = \frac{1}{sC_1} + \frac{1}{sC_2} = \frac{1}{s}\left(\frac{1}{C_1} + \frac{1}{C_2}\right) =$$

$$= \frac{1}{s\,\frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}} = \frac{1}{s\,C_{eq\,serie}}$$

$$C_{eq\,serie} = \frac{1}{\frac{1}{C_1} + \frac{1}{C_2}}$$



$$Z_{eq\,parallelo} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2}} =$$

$$= \frac{1}{sC_1 + sC_2} = \frac{1}{s(C_1 + C_2)} = \frac{1}{s\,C_{eq\,parallelo}}$$

$$C_{eq\,parallelo} = C_1 + C_2$$

$Z_L = sL$ → nel dominio di Fourier $Z_L = j\omega L$

per $\omega = 0$ $Z_L = 0$ cortocircuito ——

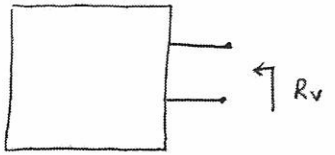
per $\omega = \infty$ $Z_L = \infty$ ramo aperto — —

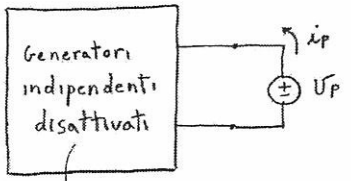
$Z_C = \frac{1}{sC}$ → nel dominio di Fourier $Z_C = \frac{1}{j\omega C}$

per $\omega = 0$ $Z_C = \infty$ ramo aperto — —

per $\omega = \infty$ $Z_C = 0$ cortocircuito ——

Resistenza vista (e in generale impedenza vista)

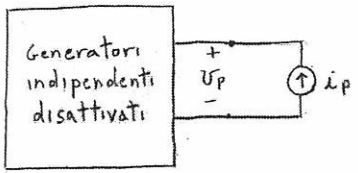




(generatori di tensione sostituiti da cortocircuiti, generatori di corrente sostituiti da rami aperti)

$$R_V = \frac{v_P}{i_P}$$

oppure



$$R_V = \frac{v_P}{i_P}$$