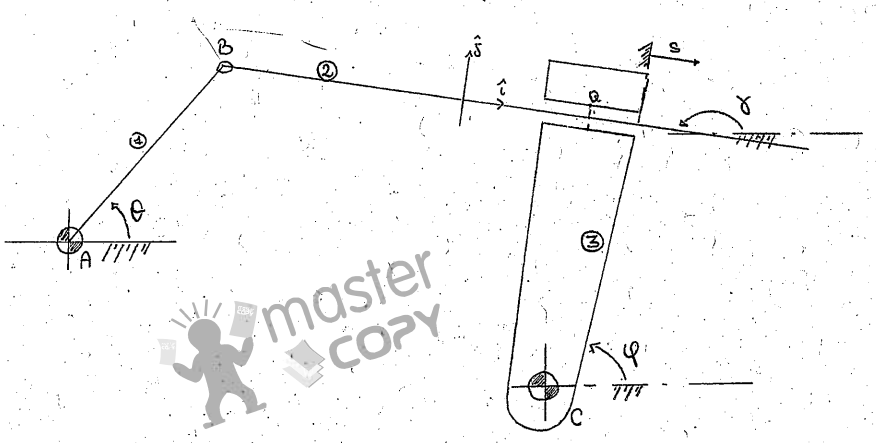
ESERCITAZIONE 1 GLIFO OSCILLANTE



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Studio della Cinematica differenziale

Atto di moto ⟶ θ, φ, γ trovo velocità $\forall$ punto di ogni corpo

2 rispetto a 1 Rototraslatorio
3 rispetto a 2 traslatorio (relativo)

Noto: θ, $\dot\theta > 0$ e $\ddot\theta > 0$

$gdl = 3\times3 - [(2\times2) + 2 + 2] = 9 - 8 = 1$

ANALISI VELOCITA'

$\vec v_A = \emptyset$

$\vec v_B = \dot\theta \hat k \wedge \vec{AB}$ → $\vec v_{Re1} = 0 + \dot\theta \hat k \wedge \vec{AB}$; $\vec v_{Re2} = \vec v_B + \dot\gamma \hat k \wedge \vec{BQ}$; $\vec v_{Re3} = 0 + \dot\varphi \hat k \wedge \vec{CR}$ } → $\dot\gamma = ?$, $\dot\varphi = ?$

$\vec v_C = \emptyset$

Moto Relativo 2-3 → Traslatorio

Considero Q e 2 e 3 successivamente

$\vec v_{Re2} = \vec v_{Re2}^{rel} + \vec v_{Re2}^{tr}$ Differenza = v^{rel}

Applico Teo. composizione Vel. Angolari

$\dot\gamma = \dot\varphi + \omega^{rel}$ (0 se solo trascinamento)

$\dot\gamma = \dot\varphi \Rightarrow \ddot\gamma = \ddot\varphi$

$$\dot\theta \hat k \wedge \vec{AB} + \dot\gamma \hat k \wedge \vec{BQ} - \dot\varphi \hat k \wedge \vec{CQ} = \dot s \hat\imath$$

Più semplice risolverlo prendendo B punto Notevole

$\vec v_{Be1} = \vec v_{Be2}$

$\dot\theta \hat k \wedge \vec{AB} = \dot s \hat\imath + \dot\varphi \hat k \wedge \vec{CB}$

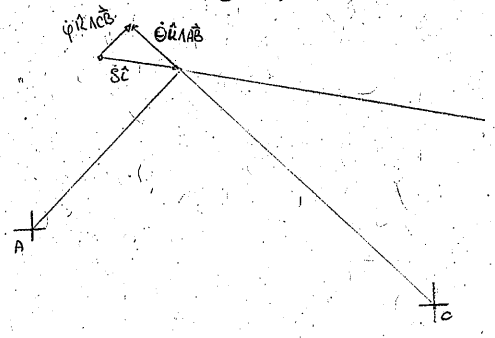
(rel) (trasc.)

↳ Eq. di Chiusura 3,2

$\dot\gamma = \dot\varphi \Rightarrow \ddot\gamma = \ddot\varphi$

$$\begin{cases} \dot\theta \hat k \wedge \vec{AB} - \dot\varphi \hat k \wedge \vec{CB} = \dot s \hat\imath \\ \ddot\gamma = \ddot\varphi \end{cases}$$

Sol. Grafica

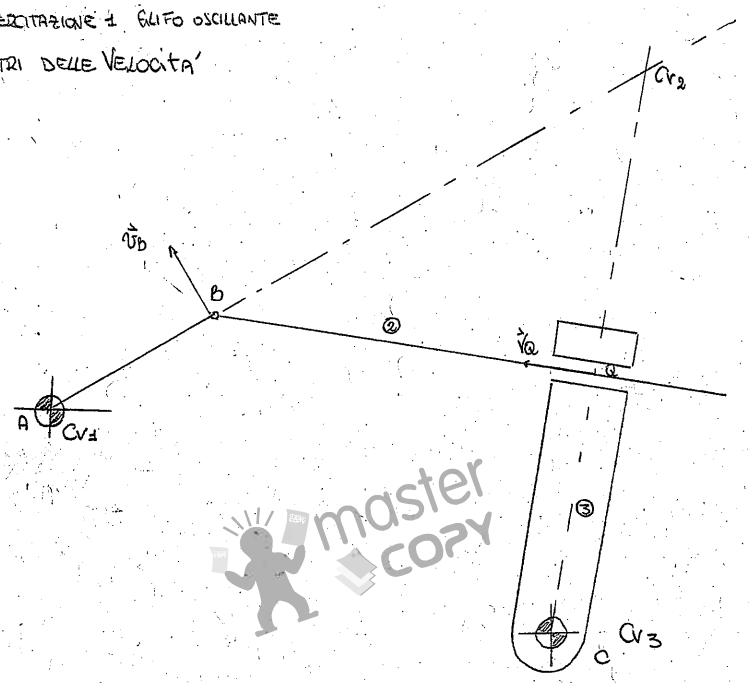


$\dot\theta > 0$

$\dot\varphi < 0$

ESERCITAZIONE 1 GUIDO OSCILLANTE

CENTRI DELLE VELOCITÀ



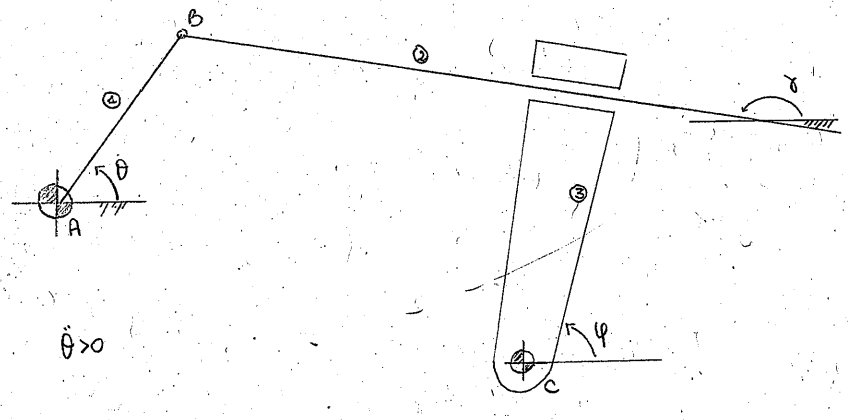
Per trovare Cv_2 -

$\in$③: $\vec{v}_{Q\in 2} = \dot{s}\cdot\vec{\imath} + \dot{\varphi}\hat{k}\wedge\vec{CQ}$

devo annullare la $\vec{v}^{trasc} = \dot{\varphi}\hat{k}\wedge\vec{CQ}$

$Q\equiv C \rightarrow \vec{v}^{trasc} = \vec{0}$

Studio Accelerazioni



$\ddot{\theta} > 0$

$\vec{a}_A = 0$

$\vec{a}_B = \ddot{\theta}\wedge\vec{AB} - \dot{\theta}^2\vec{AB}$ (Rivals)

$\vec{a}_C = 0$

$\rightarrow$ $\vec{a}_{P\in 1} = \hat{k}\ddot{\theta}\wedge\vec{AP} + \dot{\theta}^2\vec{AP}$

$\vec{a}_{Q\in 2} = \vec{a}_B + \hat{k}\ddot{\gamma}\wedge\vec{BQ} - \dot{\gamma}^2\vec{BQ}$

$\vec{a}_{r\in 3} = \vec{a}_C + \hat{k}\ddot{\varphi}\wedge\vec{CR} - \dot{\varphi}^2\vec{CR}$ (I)



B punto Notevole

$\vec{a}_{B\in 1} = \vec{a}_{B\in 2} \Rightarrow \vec{a}_{Q\in 2} = \ddot{\theta}\hat{k}\wedge\vec{AB} - \dot{\theta}^2\vec{AB} + \ddot{\gamma}\hat{k}\wedge\vec{BQ} - \dot{\gamma}^2\vec{BQ}$ (II)

Per Relazione precedente abbiamo $\ddot{\gamma} = \ddot{\varphi}$

ESERCITAZIONE 1 GLIFO OSCILLANTE

Cerco eq. di chiusura.

Osservo moto 2-3 => Traslatorio

(2) $\ddot{a}_{OE,i} = \ddot{S}\hat{\imath} + \ddot{\varphi}\hat{k}\wedge\vec{CE} - \dot{\varphi}^2\vec{CE} + 2\dot{\varphi}\hat{k}\wedge\dot{S}\hat{\imath}$

Ass. ↓ rel. ↓ trasc.

$\ddot{\theta}\hat{k}\wedge\vec{AB} - \dot{\theta}^2\vec{AB} + \ddot{\gamma}\hat{k}\wedge\vec{BE} - \dot{\gamma}^2\vec{BE} = \ddot{S}\hat{\imath} + \ddot{\varphi}\wedge\vec{CE} - \dot{\varphi}^2\vec{CE} + 2\dot{\varphi}\hat{k}\wedge\dot{S}\hat{\imath}$ → Rivalsa = Teo. completamento acc.

Dato che il moto 2-3 è di Trascinamento abbiamo $\ddot{\gamma} = \ddot{\varphi}$

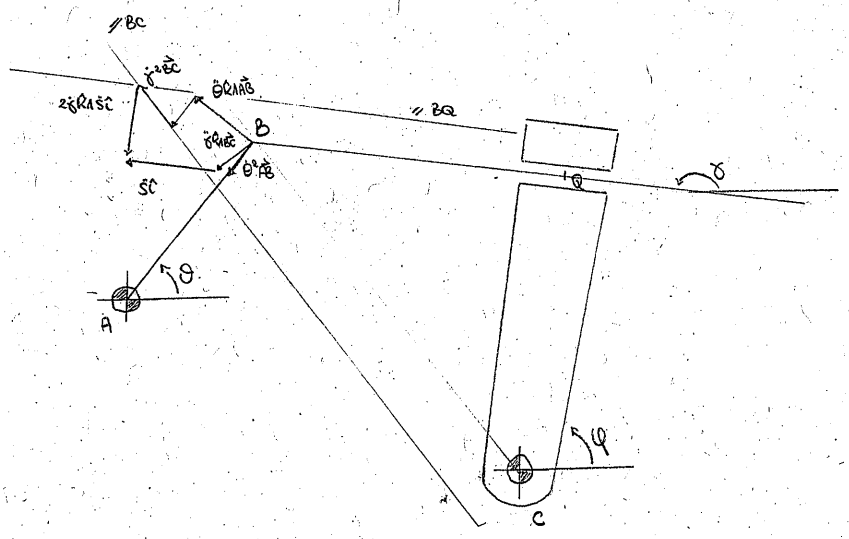
$\ddot{\theta}\hat{k}\wedge\vec{AB} - \dot{\theta}^2\vec{AB} + \ddot{\gamma}\hat{k}\wedge\vec{BE} - \dot{\gamma}^2\vec{BE} - \ddot{\gamma}\wedge\vec{CE} + \dot{\gamma}^2\vec{CE} - 2\dot{\gamma}\hat{k}\wedge\dot{S}\hat{\imath} = \ddot{S}\hat{\imath}$

$\ddot{\theta}\hat{k}\wedge\vec{AB} - \dot{\theta}^2\vec{AB} + \ddot{\gamma}\hat{k}\wedge(\vec{BC}) - \dot{\gamma}^2(\vec{BC}) - 2\dot{\gamma}\hat{k}\wedge\dot{S}\hat{\imath} = \ddot{S}\hat{\imath}$

$\ddot{\theta}$ NOTO, $\dot{\theta}$ NOTO, $\ddot{\gamma}$ DIR., $\dot{\gamma}$ NOTO, $\ddot{S}$ DIR.

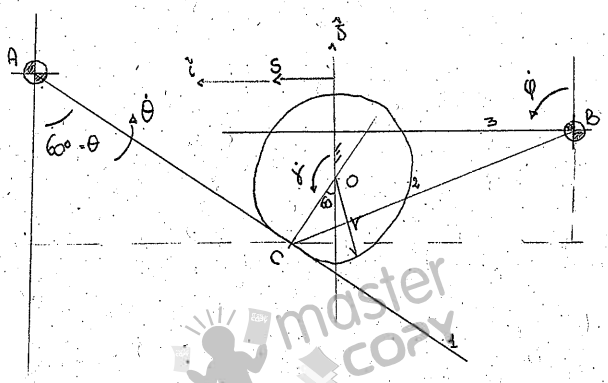
$\ddot{\theta}\hat{k}\wedge\vec{AB} - \dot{\theta}^2\vec{AB} - \dot{\gamma}^2(\vec{BC}) - 2\dot{\gamma}\hat{k}\wedge\dot{S}\hat{\imath} = \ddot{S}\hat{\imath} - \ddot{\gamma}\hat{k}\wedge\vec{BC}$ => III° Equazione

GRAFICAMENTE



$\ddot{\theta} > 0$
$\ddot{\varphi} > 0$

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Noto:
$\theta, \dot\theta > 0$ $AO = r$
r "
Iu C RSS.

Traccia
1) Vp e 1,2,3
2) Soluzione grafica di v
3) Soluzione espressioni $\bar\theta$ incognite
4) Valori per $\dot\Theta = \frac{\pi}{2}$ rad/sec e $r = 0{,}5$ m
5) C_{12}
6) Soluzione accelerazioni

2 rispetto a 1 => Rotolamento
2 rispetto a 3 => Traslazione

$P \in 1$, $Q \in 2$, $R \in 3$

$\vec v_P = \dot\theta \hat R \wedge \vec{AP}$

C è centro istantaneo di rotazione, dato che 2 rispetto a 1 ROTOLA

(1) $\vec v_Q = \vec v^{rel} + \vec v^{trasc.}$
(rot. intorno a C) $\vec v_C \equiv \vec v_{C1A}$

$\vec v_Q = \dot\gamma \hat R \wedge \vec{CQ} + \dot\theta \hat R \wedge \vec{AC}$

$\vec v_R = \dot\varphi \hat R \wedge \vec{BR}$

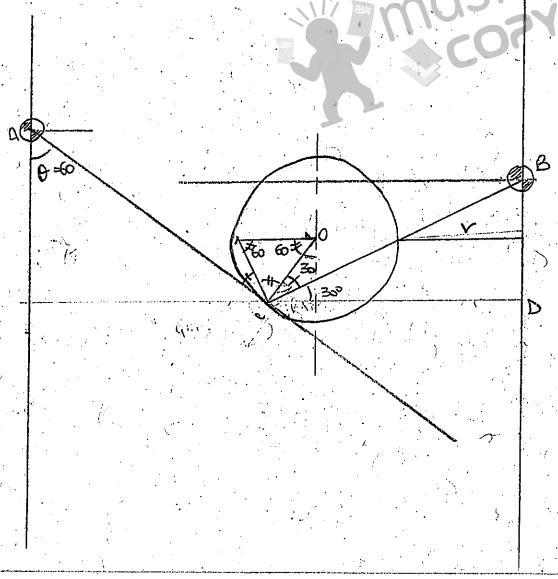
Eq. Chiusura teo. comp. Velocità Angolari
se 2 rispetto a 3 traslo

(3) $\omega_{2e2} = \omega^{rel} + \omega^{trasc} = \dot\varphi = \dot\gamma$
($\omega^{rel} = 0$)

Eq. Chiusura => Chasles = Teo. comp. velocità => Punto C

$\dot\theta \hat R \wedge \vec{AC} = \dot s \hat\imath + \dot\varphi \hat R \wedge \vec{BC}$
$\dot\theta \hat R \wedge \vec{AC}$: Chasles; $\dot s \hat\imath$: Vrel; $\dot\varphi \hat R \wedge \vec{BC}$: trasc.

$$\begin{cases} \dot\theta \hat R \wedge \vec{AC} = \dot s \hat\imath + \dot\varphi \hat R \wedge \vec{BC} \\ \dot\varphi = \dot\gamma \end{cases}$$



Il triangolo delle Velocità è Equilatero

$\dot\varphi < 0$
$\dot\gamma < 0$
$\dot s < 0$

3) Risolvo eq. con $\theta = 60°$ in funzione di $\dot{\theta}$

Trovare $\dot{\gamma}$ e $\dot{s}$

$\|\dot{s}\hat{\imath}\| = \|\dot{\varphi}\hat{k} \wedge \vec{BC}\| = \|\dot{\theta}\hat{k} \wedge \vec{AC}\|$

$\hat{\imath}) \; \dot{\theta}\hat{k} \wedge \vec{AC} \cdot \hat{\imath} = \dot{s} + \dot{\varphi}\hat{k} \wedge \vec{BC} \cdot \hat{\imath}$

$\hat{\jmath}) \; \dot{\theta}\hat{k} \wedge \vec{AC} \cdot \hat{\jmath} = \dot{\varphi}\hat{k} \wedge \vec{BC} \cdot \hat{\jmath}$

$\dot{\theta}\hat{k} \wedge \vec{AC} \cdot \cos(60°) = \dot{s} + \dot{\varphi}\hat{k} \wedge \vec{BC} \cdot \cos(60°)$

$\dot{\theta}\hat{k} \wedge \vec{AC} \cdot \text{sen}(60°) = \dot{\varphi}\hat{k} \wedge \vec{BC} \, \text{sen}(60°)$

$AC = \dfrac{r + r\,\text{sen}(60)}{\cos(60)} = r(2+\sqrt{3})$

$BC = \dfrac{2r + r\,\text{sen}(30)}{\cos(30)} = \dfrac{5\sqrt{3}}{3} r$

$$\begin{cases} \dot{\theta}\,\dfrac{2+\sqrt{3}}{2} r = \dot{s} + \dot{\varphi} r \dfrac{5\sqrt{3}}{6} \\ \dot{\theta} r \dfrac{3+2\sqrt{3}}{2} = \dot{\varphi} r \dfrac{5}{2} \end{cases} \Rightarrow \dot{\varphi} = -\dfrac{3+2\sqrt{3}}{5}\dot{\theta}$$

Verso opposto dato $\dot{\varphi} < 0$ e $\dot{\theta} > 0$

$\dot{\theta}\,\dfrac{2+\sqrt{3}}{2} r = \dot{s} + r\dot{\theta}\,\dfrac{2+\sqrt{3}}{2}$

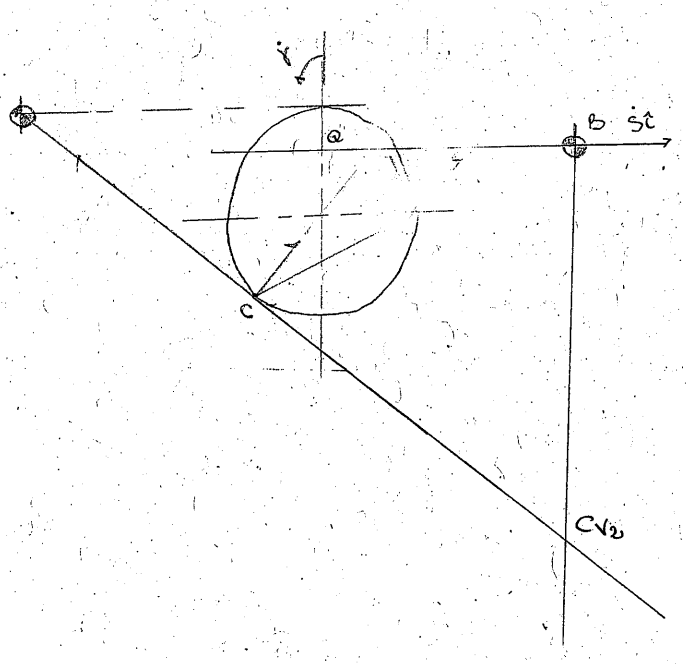
$\dot{s} = \dot{\theta} r (2+\sqrt{3})$ ✓

4) $\dot{\theta} = \frac{\pi}{2}$ rad/sec $\quad r = 0{,}5$ m

$\dot{s} = 2{,}93$ m/s

$\dot{\varphi} = -2{,}03$ rad/sec.

5) Cv_2



$v_{Q}_{e2} = \dot{s}\hat{\imath} + \dot{\varphi}\hat{k} \wedge \vec{BQ}$

Ma quando $Q \equiv B$ $v_{Qe2} = \dot{s}\hat{\imath}$