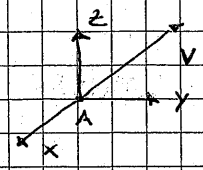
RIPASSO GENERALE

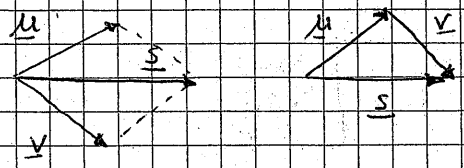
28/09/2018

VETTORE : $|V| = \sqrt{V_x^2 + V_y^2 + V_z^2}$ MODULO

$\hat{V} = \frac{V}{|V|}$ = VERSORE

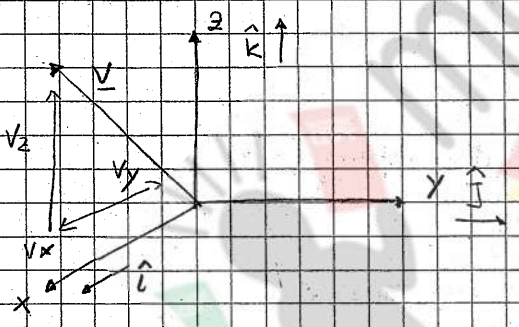


SOMMA VETTORIALE : $\underline{u} + \underline{v} = \underline{s}$ $\Rightarrow$ $\begin{bmatrix} u_x \\ u_y \\ u_z \end{bmatrix} + \begin{bmatrix} v_x \\ v_y \\ v_z \end{bmatrix} = \begin{bmatrix} s_x \\ s_y \\ s_z \end{bmatrix}$



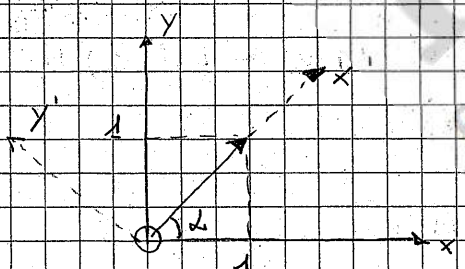
PRODOTTO : $\underline{V} = C \cdot \underline{u} = \begin{bmatrix} C \cdot u_x \\ C \cdot u_y \\ C \cdot u_z \end{bmatrix}$ $|\underline{V}| = C \cdot |\underline{u}|$

RAPPRESENTAZIONE CARTESIANA : $V = \begin{bmatrix} V_x \cdot \hat{i} \\ V_y \cdot \hat{j} \\ V_z \cdot \hat{k} \end{bmatrix}$



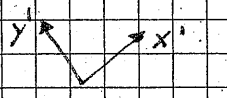
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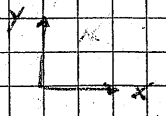
MATRICE DI ROTAZIONE : $V = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$; $|\underline{V}| = \sqrt{1^2 + 1^2}$



$$\begin{bmatrix} V_x' \\ V_y' \\ V_z' \end{bmatrix} = \begin{bmatrix} \cos\alpha & \sin\alpha & 0 \\ -\sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix}$$

SR' — Matrice di ROTAZIONE — SR





Nel nostro caso:

$$\begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ -\sqrt{2}/2 & \sqrt{2}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} = \begin{bmatrix} \sqrt{2} \\ 0 \\ 0 \end{bmatrix}$$

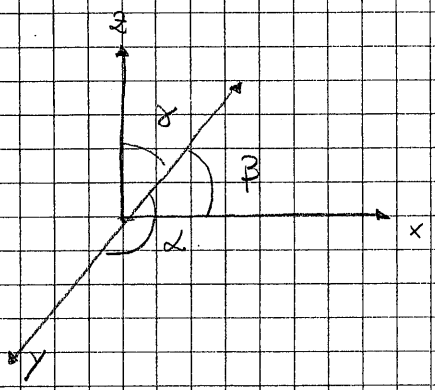
$\rightarrow$ COMPONENTI DEL NUOVO SISTEMA DI RIFERIMENTO

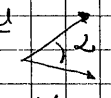
NB. IL MODULO DEL VETTORE RESTA SEMPRE UGUALE

DIFFERENZA VETTORI: $\underline{u} - \underline{v} = \underline{d} \rightarrow \underline{u} + (-\underline{v}) = \underline{d}$

PRODOTTO SCALARE: $\underline{u} \cdot \underline{v} = \overline{OV} = |\underline{u}||\underline{v}| \cos(\alpha) =$

$$[u_x \; u_y \; u_z] \begin{bmatrix} V_x \\ V_y \\ V_z \end{bmatrix} = u_x V_x + u_y V_y + u_z V_z$$





$$\hat{r} = \begin{bmatrix} \cos\alpha \\ \cos\beta \\ \cos\gamma \end{bmatrix}$$

$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$

PRODOTTO VETTORIALE: $\underline{u} \wedge \underline{v} = \underline{u} \times \underline{v} = \underline{w} \rightarrow |w| = |\underline{u}||\underline{v}| \sin\alpha$

$$\underline{w} = \begin{bmatrix} \underline{i} & \underline{j} & \underline{k} \\ u_x & u_y & u_z \\ v_x & v_y & v_z \end{bmatrix} = \dots$$

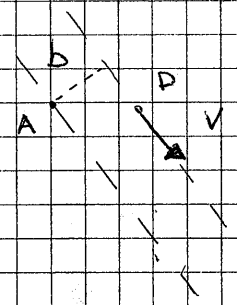
↳ per determinante ottengo $\underline{w}$

PRODOTTO MISTO: primo vettoriale poi scalare

$$\underline{u} \cdot \underline{v} \wedge \underline{w} = \begin{cases} \underline{u} \wedge \underline{v} \cdot \underline{w} \\ \underline{w} \cdot \underline{u} \wedge \underline{v} \end{cases} \quad \underline{u} \wedge (\underline{v} \wedge \underline{w})$$

$$(\underline{u} \cdot \underline{w})\underline{v} - (\underline{u} \cdot \underline{v})\underline{w}$$

MOMENTO DI UN VETTORE RISPETTO AD UN POLO



$$M_A = \overrightarrow{AP} \wedge \underline{v} = b|v|\hat{k}$$

b: braccio

SISTEMI DI VETTORI:

$$\underline{R} = \sum_{i=1}^{n} \underline{V}_i \qquad \underline{M}_O = \sum_{i=1}^{n} \overrightarrow{OP_i} \wedge \underline{V}_i$$

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Trasporto del momento da O a Ω:

$$\sum_{i=1}^{n} \overrightarrow{\Omega P_i} \wedge \underline{V}_i = \sum_{1}^{n} (\overrightarrow{\Omega O} + \overrightarrow{OP_i}) \wedge \underline{V}_i = \sum_{i=1}^{n} \overrightarrow{\Omega O} \wedge \underline{V}_i + \sum_{1}^{n} \overrightarrow{OP_i} \wedge \underline{V}_i$$

$$= \overrightarrow{\Omega O} \wedge \sum_{i=1}^{n} (\underline{V}_i) + \underline{M}_O = \overrightarrow{\Omega O} \wedge \underline{R} + \underline{M}_O$$

$$\underline{M}_\Omega = \overrightarrow{\Omega O} \wedge \underline{R} + \underline{M}_O$$

CONSIDERAZIONI: $(P_i, \underline{V}_i) \in \pi \rightarrow \begin{cases} \underline{R} \in \pi \\ \underline{M} \perp \pi \end{cases}$

$$(P_i, \underline{V}_i) = (P_i, c_i \underline{S}) \rightarrow \begin{cases} \underline{R} \parallel \underline{S} \\ \underline{M}_O \perp \underline{S} \end{cases}$$

TRINOMIO INVARIANTE: $\mathcal{I} = \underline{M}_O \cdot \underline{R} = \phi$ in tutti i sistemi piani

SISTEMA EQUIVALENTE: Due sistemi si dicono equivalenti se

$$\begin{cases} \underline{R}' = \underline{R} \\ \underline{M}_O' = \underline{M}_O \end{cases}$$

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Se $\mathcal{I} = 0 \rightarrow$ Seq $\{P, \underline{R}\}$

Se $\mathcal{I} = 0$, $\underline{R} = 0 \rightarrow$ Seq $\{M\}$

Se $\mathcal{I} \neq 0 \rightarrow$ Seq $\{(P, \underline{R}), M_{//} = \frac{\mathcal{I}}{|R|} \cdot \hat{R}\}$

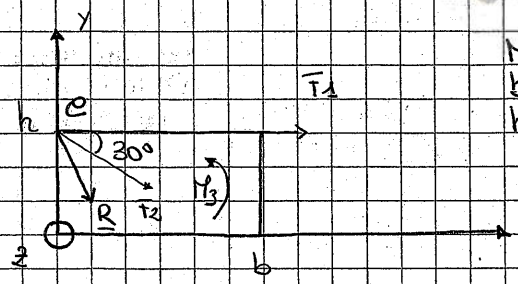
Esempio:

$$S = \{(P_1, \underline{F}_1), (P_2, \underline{F}_2), \underline{M}_3\}$$

$M_3 = 20$ Nm $\hat{k}$ $\quad \underline{F}_1 = 1$ N

$b = 10$ m $\quad \underline{F}_2 = 3$ N

$h = 4$ m $\quad$ Seq = ?



$$\underline{R} = |\underline{F}_1 + \underline{F}_2| = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + 3 \begin{bmatrix} \cos 30° \\ -\sin 30° \end{bmatrix} = \begin{bmatrix} 1 + 3\sqrt{2}/2 \\ -3/2 \end{bmatrix}$$

$\rightarrow$ Seq' = $((C, \underline{R}), \underline{M}_3)$ $\rightarrow$ mi metto in C per non avere altri momenti

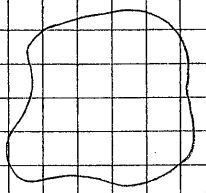
- È possibile ottenere un Seq tale che $Seq'' \{(K, \underline{R})\}$

$$\overrightarrow{KC} \wedge \underline{R} + \underline{M_3} = \underline{0}$$

$$\begin{bmatrix} X_i - X_K \\ Y_i - Y_K \\ 0 \end{bmatrix} \wedge \begin{bmatrix} 1 + 3\sqrt{3}/2 \\ -3/2 \\ 0 \end{bmatrix} = -\begin{bmatrix} 0 \\ 0 \\ 20 \end{bmatrix}$$

Allora $X_K = \frac{1}{1 + \frac{3\sqrt{3}}{20}} \left[-\frac{3}{2} X_K + 6\sqrt{3} \right]$

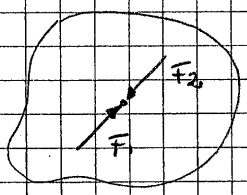
↳ RETTA ASSIALE o ASSE CENTRALE



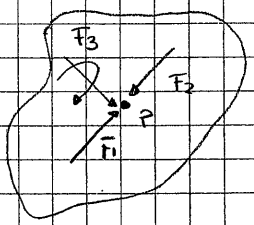
$$\begin{cases} \sum_i \underline{F_i} = m\underline{a} = \underline{0} \\ \sum_j (\underline{M_j} + \overrightarrow{AP_j} \wedge \underline{F_j}) = \underline{I}\dot{\underline{\theta}} = \underline{0} \end{cases}$$

2D	3D
$R_x = 0$	$R_x = 0$
$R_y = 0$	$R_y = 0$
	$R_z = 0$
$M_z = 0$	$M_x, M_y, M_z = 0$

Allora $\sum_i \underline{F}_{est} - \sum_j \underline{F}_{int} = \underline{0}$



COPPIA BRACCIO NULLO (condizione di equilibrio)

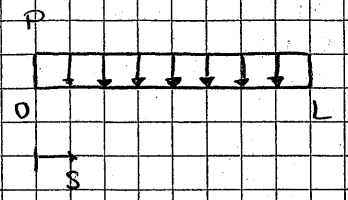


Se mi metto in P ho equilibrio del sistema

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CARICHI E VINCOLI

1) Distribuzione Rettangolare

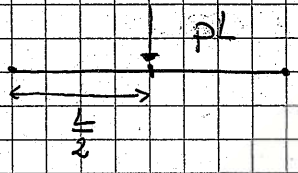


$$P = \int_0^L p(s)\,ds = p\int_0^L ds = pL$$

$$M = \int_0^L p(s)\,ds \cdot s = p\int_0^L s\,ds = \frac{pL^2}{2}$$

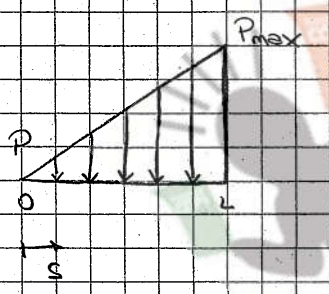
$$p = \frac{N}{m} \qquad b = \frac{M}{P} = \frac{L}{2}$$

quindi una distribuzione rettangolare è pari a:



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2) Distribuzione Triangolare



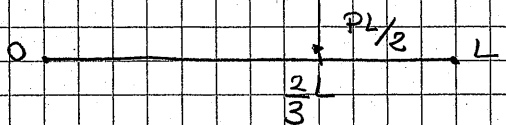
$$P = \int_0^L p(s)\,ds = \int_0^L p_{max}\frac{s}{L}\,ds = \frac{p_{max}\cdot L}{2}$$

È l'area del triangolo

$$M = \int_0^L p(s)\cdot s\,ds = \int_0^L p_{max}\frac{s}{L}\,s\,ds = \frac{p_{max}L^2}{3}$$

$$b = \frac{M}{P} = \frac{2}{3}L$$

quindi la distribuzione triangolare è pari a



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