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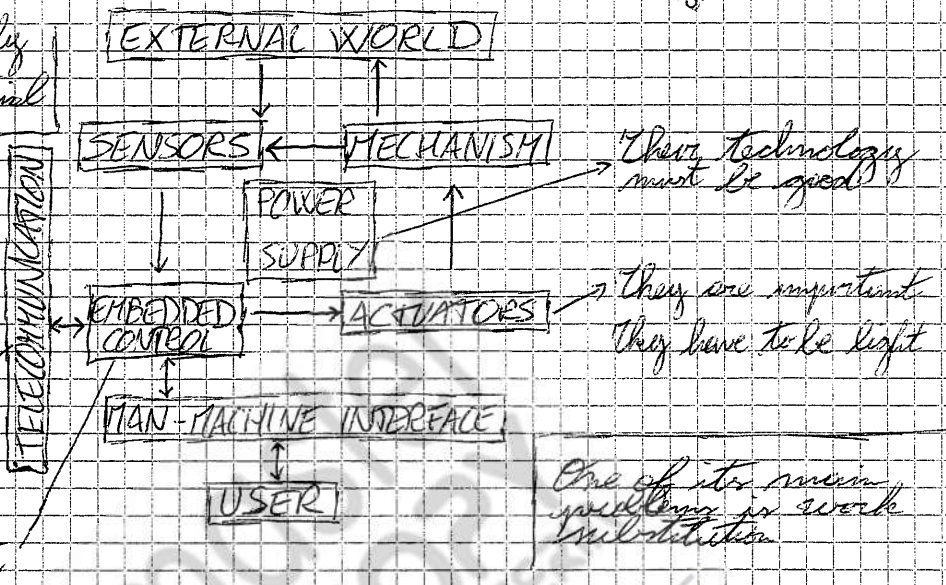
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MODEL OF A ROBOT

Robotics was initially thought for industrial purposes. Now we live in the era where robots are companions

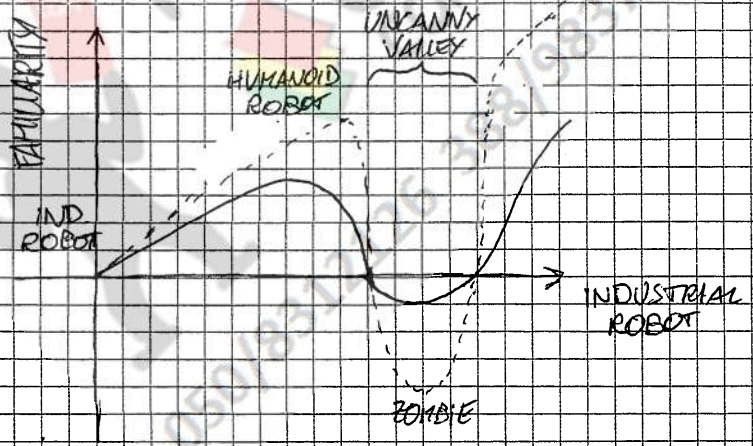
It is very useful

We have to know dynamics and mathematics of robots



MORI'S GRAPH

This graph explains that, if you propose to people something that is too human, there will be a refusal from them



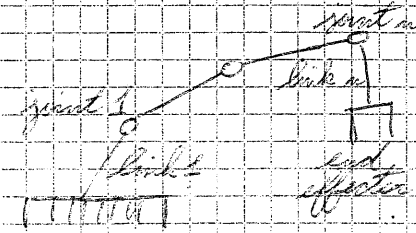
DIFFERENTIAL KINEMATICS

It provides relationship between velocity of joints and linear and angular velocities of end effector. End effector represents contact point between human and robots.

Most famous one is SCARA (Selective Compliance Assembly Robot Arm). It ~~is based on~~ composed by two rotational joints and one prismatic joint*

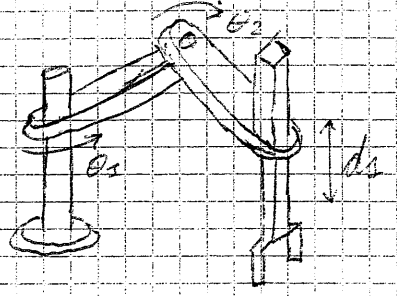

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A workspace is defined as the largest operating area of a robot that is determined by moving one joint at a time, taking into account its movement constraints.



Let us consider the manipulator to have n degrees of freedom. Be $q = [q_1, \dots, q_n]^T$ the vector of joint variables, $\dot{p}$ and ω linear and angular velocity of end effector respectively. We are now searching for these kind of relationships

$$\begin{cases} \dot{p} = f(q)\dot{q} \\ \omega = f'(q)\dot{q} \end{cases}$$



$f(q)$ and $f'(q)$ are $[3 \times n]$ matrices. $\dot{p}$ and ω can be written as

$$v = \begin{bmatrix} \dot{p} \\ \omega \end{bmatrix} = J(q)\dot{q}$$

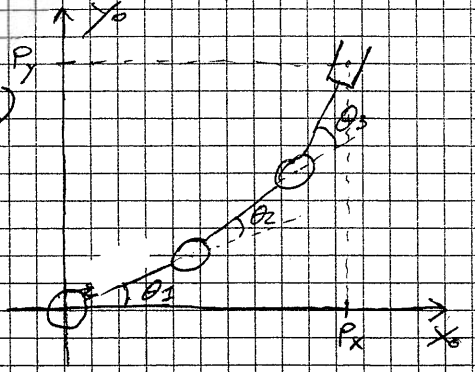
$J(q)$ is called Jacobian matrix, composed by a translational and rotational part. This is what we call a differential kinematic equation.

Considering the example

$$X = K(q) **$$

$$X = \begin{bmatrix} P_x \\ P_y \\ \phi \end{bmatrix} = \begin{bmatrix} a_1 C_1 + a_2 C_{12} + a_3 C_{123} \\ a_1 S_1 + a_2 S_{12} + a_3 S_{123} \\ \theta_1 + \theta_2 + \theta_3 \end{bmatrix}$$

$C_{12} = \cos(\theta_1 + \theta_2)$



* In rehabilitation applications, you replace the end effector with a handle.

** This is called a direct kinematics problem.

GEOMETRIC ASSESSMENT OF JACOBIAN MATRIX

The geometric Jacobian is a transformation matrix that depends on configuration of manipulator, describing relations between joint velocities and linear and angular velocities of end effector.

The computation follows geometric procedure. Jacobian can be expressed as

$$J = \begin{bmatrix} J_{P_1} & \cdots & J_{P_n} \\ J_{O_1} & \cdots & J_{O_n} \end{bmatrix}$$

where J_{P_i} and J_{O_i} are 3×1 matrices. By that, $\dot{q}_i J_{P_i}$ and $\dot{q}_i J_{O_i}$ represent contributes of joint i ~~to rotation and~~ to position and angle. Then, q_i can be a θ_i or d_i, depending on joint type.

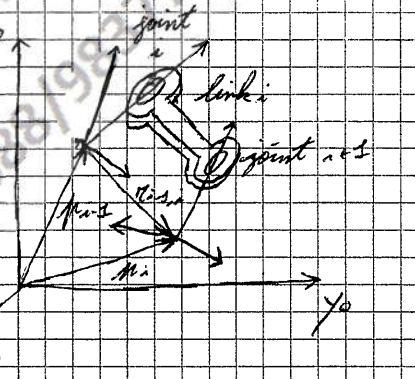
For a prismatic joint, we have

$\dot{q}_i J_{O_i} = 0 \Rightarrow J_{O_i} = 0$ (it doesn't rotate, as definition)

$\dot{q}_i J_{P_i} = \dot{d}_i z_{i-1} \Rightarrow J_{P_i} = z_{i-1}$

In this last case, we have to introduce vectors in notation

In differential kinematics, we consider a base reference system $X_0 Y_0 Z_0$. Then, for each joint i we define a reference system $i-1$ whose z_{i-1} axis is pointing to the joint i [*].



Making into account notations in figure, we can say, for a rotational joint

$\dot{q}_i J_{O_i} = \dot{\theta}_i z_{i-1} \Rightarrow J_{O_i} = z_{i-1}$

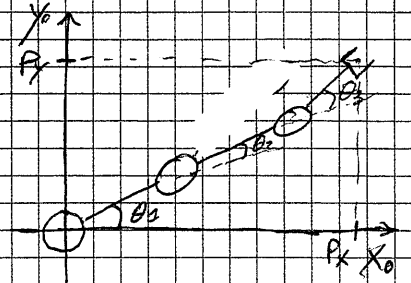
About contribution to position, we remember that $v = \omega \times r$. So,

$\dot{q}_i J_{P_i} = w_{i-1,i} \times r_{i-1,n} = \dot{\theta}_i z_{i-1} \times (p - p_{i-1}) \Rightarrow J_{P_i} = z_{i-1} \times (p - p_{i-1})$

Summing up all these results, we obtain

$$\begin{bmatrix} J_{P_i} \\ J_{O_i} \end{bmatrix} = \begin{cases} \begin{bmatrix} z_{i-1} \\ 0 \end{bmatrix} \\ \begin{bmatrix} z_{i-1} \times (p - p_{i-1}) \\ z_{i-1} \end{bmatrix} \end{cases}$$

Recalling previous example, we have



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$$p_0 = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad p_1 = \begin{bmatrix} a_1 C_1 \\ a_1 S_1 \\ 0 \end{bmatrix} \quad p_2 = \begin{bmatrix} a_1 C_1 + a_2 C_{12} \\ a_1 S_1 + a_2 S_{12} \\ 0 \end{bmatrix}$$

$$p = \begin{bmatrix} a_1 C_1 + a_2 C_{12} + a_3 C_{123} \\ a_1 S_1 + a_2 S_{12} + a_3 S_{123} \\ 0 \end{bmatrix} \quad z_0 = z_1 = z_2 = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$J(q) = \begin{bmatrix} z_0 \times (p - p_0) & z_1 \times (p - p_1) & z_2 \times (p - p_2) \\ z_0 & z_1 & z_2 \end{bmatrix}$$

$$z_0 \times (p - p_0) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} a_1 C_1 + a_2 C_{12} + a_3 C_{123} \\ a_1 S_1 + a_2 S_{12} + a_3 S_{123} \\ 0 \end{bmatrix} = \begin{bmatrix} -a_1 S_1 - a_2 S_{12} - a_3 S_{123} \\ a_1 C_1 + a_2 C_{12} + a_3 C_{123} \\ 0 \end{bmatrix}$$

$$z_1 \times (p - p_1) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} a_2 C_{12} + a_3 C_{123} \\ a_2 S_{12} + a_3 S_{123} \\ 0 \end{bmatrix} = \begin{bmatrix} -a_2 S_{12} - a_3 S_{123} \\ a_2 C_{12} + a_3 C_{123} \\ 0 \end{bmatrix}$$

$$z_2 \times (p - p_2) = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \times \begin{bmatrix} a_3 C_{123} \\ a_3 S_{123} \\ 0 \end{bmatrix} = \begin{bmatrix} -a_3 S_{123} \\ a_3 C_{123} \\ 0 \end{bmatrix}$$

$$J(q) = \begin{bmatrix} -a_1 S_1 - a_2 S_{12} - a_3 S_{123} & -a_2 S_{12} - a_3 S_{123} & -a_3 S_{123} \\ a_1 C_1 + a_2 C_{12} + a_3 C_{123} & a_2 C_{12} + a_3 C_{123} & a_3 C_{123} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 1 & 1 & 1 \end{bmatrix}$$

→ In fact, only rotation around z_0 is allowed

*It is because of how joint variables are defined

BRIEF RECAP

$$v = \begin{bmatrix} \dot{p} \\ \omega \end{bmatrix} = J(q)\dot{q} \qquad J(q) = \begin{bmatrix} J_P \\ J_O \end{bmatrix} \quad 6 \times n$$

DIFFERENTIAL KINEMATICS EQUATION

n mobility degrees

$X = K(q)$ DIRECT KINEMATIC EQUATION

↓ POSITION AND ORIENTATION OF END EFFECTOR

$$X = K(q) = \begin{bmatrix} p_x \\ p_y \\ p_z \end{bmatrix} = \begin{bmatrix} a_1 C_1 + a_2 C_{12} + a_3 C_{123} \\ a_1 S_1 + a_2 S_{12} + a_3 S_{123} \\ \theta_1 + \theta_2 + \theta_3 \end{bmatrix}$$

