RIPASSO FISICA 2

- **CAMPO SCALARE:** regione di spazio definita da una funzione scalare $U = U(P) = U(r)$
- **CAMPO VETTORIALE:** regione definita da una funzione vettoriale $\underline{U} = \underline{U}(r) = U_x(r)\hat{i}_x + U_y(r)\hat{i}_y + \dots$ definito da linee di forza del campo, tangenti in ogni punto ad esso

CAMPI → STATICI : tempo indipendenti
CAMPI → DINAMICI : tempo dipendenti

GRADIENTE DI UN CAMPO SCALARE = VETTORE

$f(x,y,z)$ campo scalare — è $\perp$ alle superfici su cui f è costante e ha dir. e verso dove il campo scalare cresce

$$\nabla f = \left(\frac{df}{dx}\hat{i}_x + \frac{df}{dy}\hat{i}_y + \frac{df}{dz}\hat{i}_z \right)$$

DIVERGENZA DI UN CAMPO VETTORIALE = SCALARE ? tendenza di un campo a convergere o divergere in un punto

$$\underline{A} = A_x\hat{i}_x + A_y\hat{i}_y + A_z\hat{i}_z$$

$$\nabla \cdot \underline{A} = \frac{dA_x}{dx} + \frac{dA_y}{dy} + \frac{dA_z}{dz}$$

$$\nabla \cdot (\nabla \times \underline{a}) = 0$$

ROTORE DI UN CAMPO VETTORIALE = VETTORE ? descrive l'irrigidimento rot di un campo vettoriale

$$\underline{A} = A_x\hat{i}_x + A_y\hat{i}_y + A_z\hat{i}_z$$

$$\nabla \times \underline{A} = \begin{vmatrix} \hat{i}_x & \hat{i}_y & \hat{i}_z \\ \frac{d}{dx} & \frac{d}{dy} & \frac{d}{dz} \\ A_x & A_y & A_z \end{vmatrix}$$

$$\nabla \times (\nabla f) = 0$$



TEOREMA DEL ROTORE (STOCKES)

$$\oint_L \underline{A} \cdot d\underline{l} = \iint_S (\nabla \times \underline{A}) \cdot d\underline{S}$$

TEOREMA DELLA DIVERGENZA (GAUSS)

$$\oiint_S \underline{A} \cdot d\underline{S} = \iiint_V (\nabla \cdot \underline{A})\, dV$$

LAPLACIANO DI UN CAMPO SCALARE = SCALARE

$f(x,y,z)$ scalare — $\nabla^2 f = \nabla \cdot (\nabla f)$ div (grad f)

$$\nabla^2 f = \frac{d^2f}{dx^2} + \frac{d^2f}{dy^2} + \frac{d^2f}{dz^2}$$

FASORE

Segnale armonico nel $t \rightarrow a(t) = A_M \cos(\omega t + \varphi)$

Fasore $A = A_M e^{j\varphi}$ da cui torno in t con $a(t) = \mathrm{Re}\{A_M e^{j\omega t}\} = \mathrm{Re}\{A e^{j(\varphi + \omega t)}\}$

Se ho un campo vettoriale $\underline{e}(t) = E_{xM}\cos(\omega t + \varphi_x)\hat{x} + E_{yM}\cos(\omega t + \varphi_y)\hat{y} + E_{zM}\cos(\omega t + \varphi_z)\hat{z}$

Fasore $\underline{E} = E_{xM} e^{j\varphi_x}\hat{x} + E_{yM} e^{j\varphi_y}\hat{y} + E_{zM} e^{j\varphi_z}\hat{z}$

$\hookrightarrow \underline{e}(t) = \mathrm{Re}\{\underline{E} e^{j\omega t}\} = \underline{A}\cos(\omega t) - \underline{B}\,\mathrm{sen}(\omega t)$

EQUAZIONI DI MAXWELL

4 equazioni differenziali in 4 variabili che descrivono le interazioni tra $\underline{E}$ e $\underline{H}$
- 2 descrivono la generazione del campo dalle cariche
- 2 come il campo circola attorno alle sorgenti

> Dominio t → FORMA DIFFERENZIALE

1) $\nabla \times \underline{e}(\underline{r},t) = -\frac{d}{dt}\underline{b}(\underline{r},t)$
2) $\nabla \times \underline{h}(\underline{r},t) = \frac{d}{dt}\underline{d}(\underline{r},t) + \underline{j}(\underline{r},t)$
3) $\nabla \cdot \underline{d}(\underline{r},t) = \rho(\underline{r},t)$
4) $\nabla \cdot \underline{b}(\underline{r},t) = 0$

$\underline{e}$ [V/m] $\quad \underline{h}$ [A/m]
$\underline{b}$ [T/m²] $\quad \underline{d}$ [C/m²]
ρ [C/m³] $\quad \underline{j}$ [A/m²]

Non sono equazioni indipendenti

① → ④

$\nabla \times \underline{e} = -\frac{d}{dt}\underline{b}(\underline{r},t)$ moltiplico per div entrambi i membri

$\nabla \cdot (\nabla \times \underline{e}) = -\nabla \cdot \left[\frac{d}{dt}\underline{b}(\underline{r},t)\right]$ ma $\nabla \cdot (\nabla \times \underline{a}) = 0$

$\nabla \cdot \frac{d\underline{b}}{dt} = 0 \rightarrow \frac{d}{dt}(\nabla \cdot \underline{b}) = 0$ $\quad \nabla \cdot \underline{b}$ costante nel tempo

$t < t_0$ sorgenti "spente" $\quad \underline{b}(t<t_0) = 0 \quad \nabla \cdot \underline{b} = 0 \quad \forall t$

② → ③

$\nabla \times \underline{h}(\underline{r},t) = \frac{d}{dt}\underline{d}(\underline{r},t) + \underline{j}(\underline{r},t)$ moltiplico per div

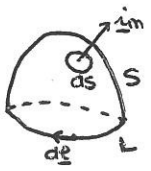
$\nabla \cdot [\nabla \times \underline{h}(\underline{r},t)] = \nabla \cdot \frac{d}{dt}\underline{d}(\underline{r},t) + \nabla \cdot \underline{j}(\underline{r},t)$ ma $\nabla \cdot (\nabla \times \underline{a}) = 0$

$\nabla \cdot \frac{d}{dt}\underline{d}(\underline{r},t) + \nabla \cdot \underline{j}(\underline{r},t) = 0 \qquad \nabla \cdot \underline{j}(\underline{r},t) + \frac{d}{dt}[\nabla \cdot \underline{d}(\underline{r},t)] = 0$

$\nabla \cdot \underline{d}(\underline{r},t) = \rho(\underline{r},t)$ ③

$\boxed{\nabla \cdot \underline{j} + \frac{d}{dt}\rho = 0}$ **EQUAZIONE DI CONTINUITÀ**

> Dominio t → FORMA INTEGRALE



Calcolo il flusso di 1ª e 2ª attraverso S con vers $\underline{i}_m$ che poggia su L con vers $\underline{i}_\ell$ (S e L fissi nel t)

1) $\iint_S \nabla \times \underline{e} \cdot \underline{i}_m \, dS = -\iint_S \frac{d\underline{b}}{dt} \cdot \underline{i}_m \, dS$

2) $\iint_S \nabla \times \underline{h} \cdot \underline{i}_m \, dS = \iint_S \frac{d\underline{d}}{dt} \cdot \underline{i}_m \, dS + \iint_S \underline{j} \cdot \underline{i}_m \, dS$

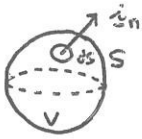
DAL TEOREMA DI STOKES O DEL ROTORE $\quad \oint_L \underline{A} \cdot d\underline{\ell} = \iint_S \nabla \times \underline{A} \cdot \underline{i}_m \, dS$

1) $\oint_L \underline{e} \cdot \underline{i}_\ell \cdot d\ell = -\frac{d}{dt}\iint_S \underline{b} \cdot \underline{i}_m \, dS$ — Legge di Maxwell-Faraday

2) $\oint_L \underline{h} \cdot \underline{i}_\ell \cdot d\ell = \frac{d}{dt}\iint_S \underline{d} \cdot \underline{i}_m \, dS + \iint_S \underline{j} \cdot \underline{i}_m \, dS$ — Generalizzazione della legge di Ampère

3) $\iiint_V (\nabla \cdot \underline{d})\, dV = \iiint_V \rho\, dV$

4) $\iiint_V \nabla \cdot \underline{b} \cdot dV = 0$



Volume V delimitato da superficie S fissi nel t

DAL TEOREMA DI GAUSS O DELLA DIVERGENZA $\iint_S \underline{A} \cdot \underline{i}_m\, dS = \iiint_V \nabla \cdot \underline{A}\, dV$

3) $\oint\!\!\oint_S \underline{d} \cdot \underline{i}_m\, dS = \iiint_V \rho\, dV$ — Legge di Gauss

4) $\oint\!\!\oint_S \underline{b} \cdot \underline{i}_m\, dS = 0$ — Gauss per il magnetismo

! La legge di Gauss $\oint\!\!\oint_S \underline{E} \cdot d\underline{S} = \frac{Q}{\varepsilon_0}$ viene dalla 3ª di Maxwell

Dall'equazione di continuità $\nabla \cdot \underline{j} + \frac{d}{dt}\rho = 0$ integro su V

$\iiint_V (\nabla \cdot \underline{j})\, dV = -\iiint_V \frac{d\rho}{dt}\, dV$ dal teo di Gauss $\oint\!\!\oint_S \underline{A} \cdot \underline{i}_m\, dS = \iiint_V \nabla \cdot \underline{A}\, dV$

$$\oint\!\!\oint_S \underline{j} \cdot \underline{i}_m\, dS = -\frac{d}{dt} \iiint_V \rho\, dV$$

CONSERVAZIONE DELLA CARICA = forma differenziale dell'eq. di continuità

2) $\oint_L \underline{h} \cdot \underline{i}_e\, dl = \frac{d}{dt} \iint_S \underline{d} \cdot \underline{i}_m\, dS + \iint_S \underline{j} \cdot \underline{i}_m\, dS$

pongo $\Phi(t) = \iint_S \underline{d}(\underline{r},t) \cdot \underline{i}_m\, dS$

$i(t) = \oint\!\!\oint_S \underline{j}(\underline{r},t) \cdot \underline{i}_m\, dS$

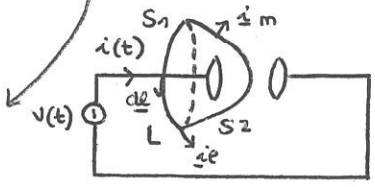
$\oint_L \underline{h} \cdot \underline{i}_e\, dl = \frac{d}{dt}\Phi(t) + i(t)$ per campi statici $\frac{d}{dt}\Phi(t) = 0$

$$i(t) = \oint_L \underline{h} \cdot \underline{i}_e\, dl$$

LEGGE DI AMPÈRE

Incoerente per i campi dinamici

utilizzando la forma differenziale per campi statici $\nabla \times \underline{h}(\underline{r},t) = \underline{j}(\underline{r},t)$



$\oint_L \underline{h}(\underline{r},t) \cdot \underline{i}_e\, dl = \iint_{S_1} \underline{j}(\underline{r},t) \cdot \underline{i}_m\, dS = i(t)$

$\oint_L \underline{h}(\underline{r},t) \cdot \underline{i}_e\, dl = \iint_{S_2} \underline{j}(\underline{r},t) \cdot \underline{i}_m\, dS \neq 0$

$$\oint_L \underline{h}(\underline{r},t) \cdot \underline{i}_e\, dl = \frac{d}{dt} \iint_S \underline{d} \cdot \underline{i}_m\, dS + \iint_S \underline{j} \cdot \underline{i}_m\, dS$$

LEGGE DI AMPÈRE GENERALIZZATA

DIFFERENZIALE

> Dominio $\mathfrak{f}$ → FORMA INTEGRALE

1) $\nabla \times \underline{E}(\underline{r},\omega) = -j\omega \underline{B}(\underline{r},\omega)$

2) $\nabla \times \underline{H}(\underline{r},\omega) = j\omega \underline{D}(\underline{r},\omega) + \underline{J}(\underline{r},\omega)$

3) $\nabla \cdot \underline{D}(\underline{r},\omega) = \rho(\underline{r},\omega)$

4) $\nabla \cdot \underline{B}(\underline{r},\omega) = 0$

$\nabla \cdot \underline{J}(\underline{r},\omega) + j\omega\rho(\underline{r},\omega) = 0$

> Dominio $\mathfrak{F}$ → FORMA INTEGRALE

1) $\oint_L \underline{E} \cdot \underline{i}_e \cdot dl = -j\omega \iint_S \underline{B} \cdot \underline{i}_m\, dS$

2) $\oint_L \underline{H} \cdot \underline{i}_e \cdot dl = j\omega \iint_S \underline{D} \cdot \underline{i}_m dS + \iint_S \underline{j} \cdot \underline{i}_m dS$

3) $\oint\!\!\oint_S \underline{D} \cdot \underline{i}_m dS = \iiint_V \rho\, dV$

4) $\oint\!\!\oint_S \underline{B} \cdot \underline{i}_m\, dS = 0$

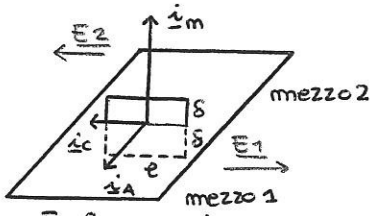
$\oint\!\!\oint_S \underline{J} \cdot \underline{i}_m dS = j\omega \iiint_V \rho\, dV$

CONDIZIONI AL CONTORNO

Le ricavo senza avere info sul materiale e spiegamo una componente su due

Hp ℓ, δ piccole così da poter considerare i campi costanti nell'intorno

- terna ortogonale $\underline{i}_c \times \underline{i}_A = \underline{i}_m$



$\underline{E}$ lungo $\underline{i}_c$

- 1) $\oint_L \underline{e} \cdot \underline{i}_\ell \cdot d\ell = -\frac{d}{dt} \iint_S \underline{b} \cdot \underline{i}_m \, dS$ forma integrale, dominio t

guardo le componenti tangenziali (quelle normali si eludono)

$(\underline{e}_2 \cdot \underline{i}_c)\ell + (\underline{e}_1 \cdot (-\underline{i}_c))\ell = -\frac{d}{dt}(\ell\delta \underline{b}_1 \cdot \underline{i}_A) - \frac{d}{dt}(\ell\delta \underline{b}_2 \cdot \underline{i}_A)$

→ $\underline{b}$ lungo $\underline{i}_A$ $\quad \underline{B}$ lungo $\underline{i}_A = \underline{H} = \underline{J}$

Se $\delta \to 0$ $\quad \underline{i}_c \cdot \underline{e}_2 = \underline{i}_c \cdot \underline{e}_1 \quad \forall \underline{i}_c$ $\quad \underline{e}_2 \cdot (\underline{i}_A \times \underline{i}_m) = \underline{e}_1 \cdot (\underline{i}_A \times \underline{i}_m) \leftrightarrow \underline{i}_A \cdot (\underline{i}_m \times \underline{e}_2) = \underline{i}_A \cdot (\underline{i}_m \cdot \underline{e}_1)$

$(\underline{i}_m \times \underline{e}_2) = (\underline{i}_m \times \underline{e}_1)$

$\boxed{(\underline{e}_2)_{tg} = (\underline{e}_1)_{tg}}$

- 2) $\oint_L \underline{h} \cdot \underline{i}_e \cdot d\ell = \frac{d}{dt} \iint_S \underline{d} \cdot \underline{i}_m \, dS + \iint_S \underline{j} \cdot \underline{i}_m \, dS$ sotto le stesse Hp

Guardo $\underline{h}_{tg}$

$\underline{i}_A \cdot (\underline{i}_m \times (\underline{h}_2 - \underline{h}_1)) = \lim_{\delta \to 0} (\underline{j} \cdot \underline{i}_A)\delta \quad \forall \underline{i}_A \quad \to \boxed{\underline{i}_m \times (\underline{h}_2 - \underline{h}_1) = \underline{j}_S}$

($\underline{i}_m \parallel \underline{i}_A$, ambi $\perp \underline{i}_A$)

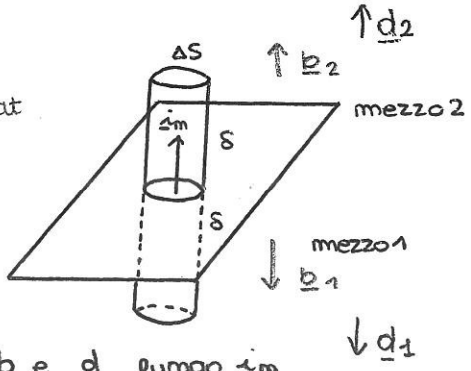
$\underline{h}$ lungo $\underline{i}_A$
$\underline{d}$ lungo $\underline{i}_c$
$\underline{J}$ lungo $\underline{i}_A$

→ $\exists$ I superficiali → $\underline{e}_{tg}$ conservata, $\underline{h}_{tg}$ discontinuita'

→ $\nexists$ I superficiali → $\underline{e}_{tg}$ conservata, $\underline{h}_{tg}$ conservata

- 4) $\oint_S \underline{b} \cdot \underline{i}_m \, dS = 0$ sotto le stesse Hp

Componenti sup tat si eludono



$\underline{b}$ e $\underline{d}$ lungo $\underline{i}_m$

$\underline{b}_2 \cdot \underline{i}_m \Delta S - \underline{b}_1 \cdot \underline{i}_m \Delta S = 0 \quad \to \quad \underline{b}_2 \cdot \underline{i}_m = \underline{b}_1 \cdot \underline{i}_m$

$\boxed{(\underline{b}_2)_{norm} = (\underline{b}_1)_{norm}}$

→ $\exists$ I superficiali → $\underline{b}_{norm}$ conservata, $\underline{d}_{norm}$ discontinua

→ $\nexists$ I superficiali → $\underline{b}_{norm}$ conservata, $\underline{d}_{norm}$ conservata

$\underline{i}_m \times (\underline{h}_2 - \underline{h}_1) = \underline{J}_S$
$\underline{i}_m \cdot (\underline{d}_2 - \underline{d}_1) = \rho$

- 3) $\oiint_S \underline{d} \cdot \underline{i}_m \, dS = \iiint_V \rho \, dV$

$\underline{i}_m \cdot \underline{d}_2 \cdot \Delta S - \underline{i}_m \underline{d}_1 \Delta S = \lim_{\delta \to 0} \rho \delta \Delta S = \rho_S \quad \to \quad \underline{i}_m (\underline{d}_2 - \underline{d}_1) = \rho_S$

master COPY COPISTERIA

ONDE PIANE soluzioni delle equazioni di Maxwell

RELAZIONI COSTITUTIVE specifiche per ogni mezzo (che invece non è considerato nelle eq. di Maxwell)

$$\begin{cases} \underline{D}(\underline{r},\omega) = \underline{\underline{\varepsilon}}(\underline{r},\omega)\underline{E}(\underline{r},\omega) + \underline{\underline{\xi}}(\underline{r},\omega)\underline{H}(\underline{r},\omega) \\ \underline{B}(\underline{r},\omega) = \underline{\underline{\zeta}}(\underline{r},\omega)\underline{E}(\underline{r},\omega) + \underline{\underline{\mu}}(\underline{r},\omega)\underline{H}(\underline{r},\omega) \end{cases} \xrightarrow{\text{le uso}} \begin{cases} \underline{d}(\underline{r},t) = \varepsilon_0 \underline{e}(\underline{r},t) \\ \underline{b}(\underline{r},t) = \mu_0 \underline{h}(\underline{r},t) \end{cases}$$

DOMINIO t

Hp 1: spazio libero, no cariche nè correnti (campo lontano), mezzo lineare, omogeneo, isotropo, non dispersivo

$$\begin{cases} \underline{d}(\underline{r},t) = \varepsilon_0 \underline{e}(\underline{r},t) \\ \underline{b}(\underline{r},t) = \mu_0 \underline{h}(\underline{r},t) \\ \nabla \times \underline{e}(\underline{r},t) = -\frac{d}{dt}\mu_0 \underline{h}(\underline{r},t) \\ \nabla \times \underline{h}(\underline{r},t) = \frac{d}{dt}\varepsilon_0 \underline{e}(\underline{r},t) \\ \nabla \cdot [\varepsilon_0 \underline{e}(\underline{r},t)] = 0 \\ \nabla \cdot [\mu_0 \underline{h}(\underline{r},t)] = 0 \end{cases}$$

{Regione di campo lontano $\rho = 0$ $J = 0$}

[redacted]

Hp 2: $\underline{E}$, $\underline{H}$ dipendenti da z $\quad \underline{e}(\underline{r},t) = \underline{e}(z,t) \quad \underline{h}(\underline{r},t) = \underline{h}(z,t)$

$$\nabla \times \underline{e} = \begin{vmatrix} \hat{i}_x & \hat{i}_y & \hat{i}_z \\ 0 & 0 & d/dz \\ e_x & e_y & e_z \end{vmatrix} = -\frac{de_y}{dz}\hat{i}_x + \frac{de_x}{dz}\hat{i}_y$$

$$\nabla \times \underline{h} = \begin{vmatrix} \hat{i}_x & \hat{i}_y & \hat{i}_z \\ 0 & 0 & d/dz \\ h_x & h_y & h_z \end{vmatrix} = -\frac{dh_y}{dz}\hat{i}_x + \frac{dh_x}{dz}\hat{i}_y$$

$$\rightarrow \begin{cases} \nabla \times \underline{e}(z,t) = -\mu_0 \frac{d}{dt}\underline{h}(z,t) \\ \nabla \times \underline{h}(z,t) = \varepsilon_0 \frac{d}{dt}\underline{e}(z,t) \end{cases}$$

$$\begin{cases} -\frac{de_y}{dz}\hat{i}_x + \frac{de_x}{dz}\hat{i}_y = \mu_0 \frac{d}{dt}(h_x\hat{i}_x + h_y\hat{i}_y + h_z\hat{i}_z) \\ -\frac{dh_y}{dz}\hat{i}_x + \frac{dh_x}{dz}\hat{i}_y = \varepsilon_0 \frac{d}{dt}(e_x\hat{i}_x + e_y\hat{i}_y + e_z\hat{i}_z) \end{cases}$$

2 eq vettoriali

$$① \begin{cases} \frac{de_y}{dz} = \mu_0 \frac{dh_x}{dt} \\ \frac{dh_x}{dz} = \varepsilon_0 \frac{de_y}{dt} \end{cases} \qquad ② \begin{cases} \frac{de_x}{dz} = -\mu_0 \frac{dh_y}{dt} \\ -\frac{dh_y}{dz} = \varepsilon_0 \frac{de_x}{dt} \end{cases}$$

con $\frac{de_z}{dt} = 0$ & $\frac{dh_z}{dt} = 0$

e_z e h_z costanti → le considero nulle
(campi statici, per le altre componenti invece sono campi dinamici)

$$\frac{d}{dz}\begin{cases} \frac{d^2e_x}{dz^2} = -\mu_0 \frac{d^2h_y}{dt\,dz} \\ -\frac{d^2h_y}{dt\,dz} = \varepsilon_0 \frac{d^2e_x}{dt^2} \end{cases} \rightarrow \frac{d^2e_x}{dz^2} = \frac{1}{c^2}\frac{d^2e_x}{dt^2}$$

$$\frac{d}{dt}\begin{cases} \frac{d^2e_x}{dt\,dz} = -\mu_0 \frac{d^2h_y}{dt^2} \\ -\frac{d^2h_y}{dz^2} = \varepsilon_0 \frac{d^2e_x}{dt\,dz} \end{cases} \rightarrow \frac{d^2h_y}{dz^2} = \frac{1}{c^2}\frac{d^2h_y}{dt^2}$$

[redacted] $\left[\frac{m}{s}\right]$

EQ. DI D'ALAMBERT

$$\frac{d^2e_x}{dz^2} - \frac{1}{c^2}\frac{d^2e_x}{dt^2} = 0 \qquad \frac{d^2h_y}{dz^2} - \frac{1}{c^2}\frac{d^2h_y}{dt^2} = 0$$

(!) Simili alle eq dei telegrafisti nella LT

Eq. diff di 2° grado hanno come soluzione la somma di 2 funzioni

$$\begin{cases} e_x(z,t) = f_1(z-ct) + f_2(z+ct) \\ h_y(z,t) = f_3(z-ct) + f_4(z+ct) \end{cases}$$

$z > 0$ $\quad$ $z < 0$

pongo $u = z - ct$ → $\frac{du}{dt} = -c$, $\frac{du}{dz} = 1$