Elements of Gaskinetic Theory

(see Bittencourt, chapters 5 and 6)

Gaskinetic theory (GKT) is a ***microscopic*** approach to study fluid dynamic and plasma dynamic problems.

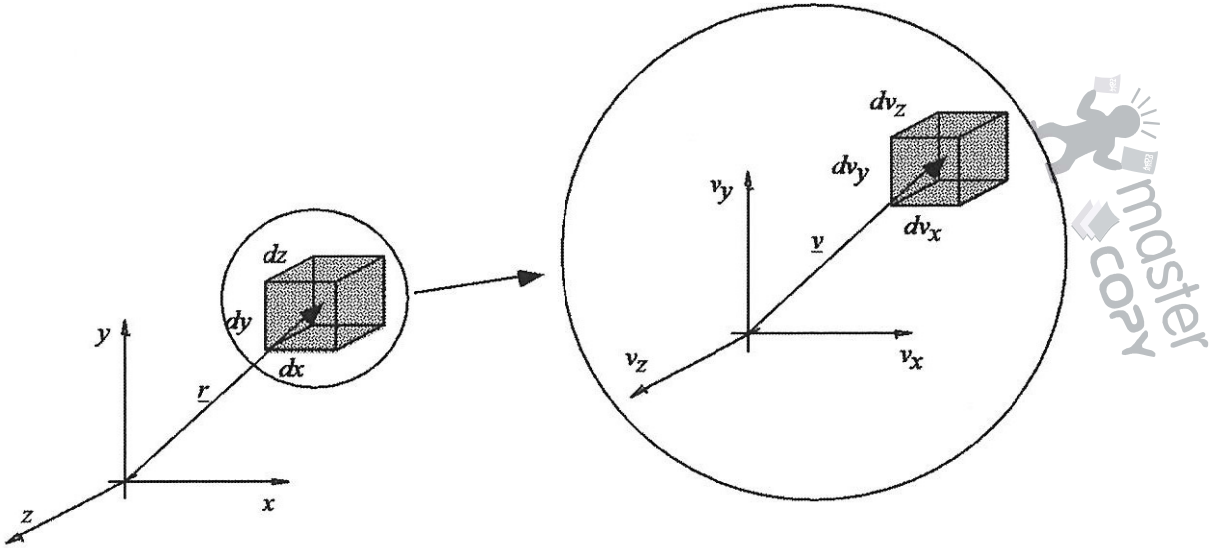
The objective of GKT is to explain and predict the ***macroscopic properties*** of gases (both neutral and ionized) from the properties of their microscopic constituents (molecules, atoms, ions, electrons).

The fundamental hypothesis of GKT is that gases are composed by a very large number of particles which are in various states of motion.

GKT takes into account the details of the interactions among the particles (for instance collisions between molecules or atoms, etc.).

Classical mechanics (**F**=m**a**) is adopted to describe the motion of the particles (except in some cases concerning the short-range interaction between particles where quantum mechanics is requested).

Let us introduce the ***phase space***: six dimensions + time: x, y, z, v_x, v_y, v_z, t



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Number density:

$$n(\vec{r},t) = \lim_{\Delta^3 r \to d^3 r} \frac{\Delta^3 N}{\Delta^3 r}$$

i.e., the number of particles per unite volume

Distribution function of velocity (DFV):

$$f(\vec{r},\vec{v},t) = \lim_{\substack{\Delta^3 r \to \mathrm{d}^3 r \\ \Delta^3 v \to \mathrm{d}^3 v}} \frac{\Delta^6 N}{\Delta^3 r \Delta^3 v}$$

At any instant of time each particle in the plasma can be localized by a position vector $\mathbf{r}$ drawn from the origin of a coordinate system to the center of mass of the particle. In a Cartesian frame of reference, as illustrated in Fig. 1, we have

$$\mathbf{r} = x\hat{\mathbf{x}} + y\hat{\mathbf{y}} + z\hat{\mathbf{z}} \tag{2.1}$$

where $\hat{\mathbf{x}}$, $\hat{\mathbf{y}}$, and $\hat{\mathbf{z}}$ denote unit vectors along the axes x, y, and z, respectively. The linear velocity of the center of mass of the particle can be represented by the vector

$$\mathbf{v} = v_x\hat{\mathbf{x}} + v_y\hat{\mathbf{y}} + v_z\hat{\mathbf{z}} \tag{2.2}$$

with $v_x = dx/dt$, $v_y = dy/dt$, and $v_z = dz/dt$.

In analogy with the *configuration space* defined by the position coordinates (x, y, z), it is convenient to introduce the *velocity space* defined by the velocity coordinates (v_x, v_y, v_z). In this space the velocity vector $\mathbf{v}$ can be viewed as a position vector drawn from the origin of the coordinate system (v_x, v_y, v_z) to the center of mass of the particle, as indicated schematically in Fig. 1.

From the point of view of classical mechanics the instantaneous *dynamic state* of each particle can be specified by its position and velocity vectors. It is convenient, therefore, to consider the *phase space* defined by the six coordinates (x, y, z, v_x, v_y, v_z).

In this six-dimensional space the dynamic state of each particle is appropriately represented by a single point. The coordinates $(\mathbf{r}, \mathbf{v})$ of the representative point give the position and velocity of the particle. When the particle moves, its representative point describes a trajectory in phase space. At each instant of time the dynamic state of a system of N particles is represented by N points in phase space.

A small element of volume in configuration space is represented by $d^3r = dx\ dy\ dz$. This differential element of volume should not be taken literally as a mathematically infinitesimal quantity but as a finite element of volume, sufficiently large to contain a very large number of particles, yet sufficiently small in comparison with the characteristic lengths associated with the spatial variation of physical parameters of interest such as, for example, density and temperature. In a gas containing 10^{18} molecules/m^3,

In case of a system made of a mixture of more species (like a plasma) we define a number density (n_α) and a DFV for each species (f_α).

It is easy to show the mixture number density and DFV are given by:

$$n = \sum n_\alpha \qquad f = \sum f_\alpha$$

Example: Simple plasma (a plasma made of electrons, single charged ions and neutrals (neutri))

$$n = n_e + n_i + n_n \qquad f = f_e + f_i + f_n$$

Let's found now the link between microscopic and macroscopic quantities.

From now we will use indifferently the terms ***particles*** and ***molecules*** as synonymous,