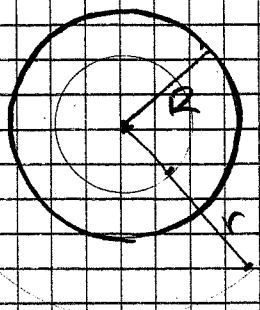
Simmetria Sferica

Sfera uniformemente carica



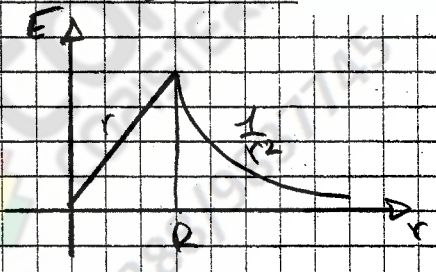
$$\Phi = E_r 4\pi r^2$$

Devo distinguere i casi in cui cerco il campo elettrico dentro e fuori la sfera.

$r > R \quad E_r 4\pi r^2 = \frac{1}{\varepsilon} \rho \frac{4}{3}\pi R^3 \quad E_r = \frac{\rho R^3}{3\varepsilon} \cdot \frac{1}{r^2}$

$r < R \quad E_r 4\pi r^2 = \frac{1}{\varepsilon} \rho \frac{4}{3}\pi r^3 \quad E_r = \frac{\rho}{3\varepsilon} r$

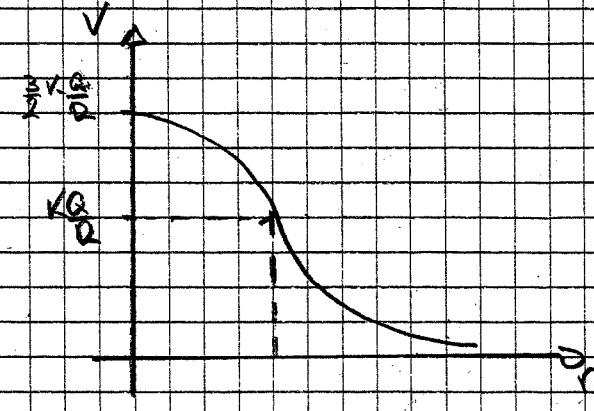
$$E_r = \begin{cases} \frac{\rho R^3}{3\varepsilon_0} \cdot \frac{1}{r^2} & r > R \\ \frac{\rho}{3\varepsilon_0} \cdot r & r < R \end{cases}$$



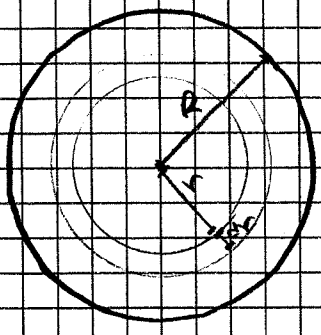
MASTER COPY
Tel. 388/9837745

$$V(r) = \int_r^{\infty} E_r dr$$

$$V(r) = \begin{cases} \frac{KQ}{r} & r \geq R \\ \frac{KQ}{2R}\left(3 - \frac{r^2}{R^2}\right) & r < R \end{cases}$$



Cipolla

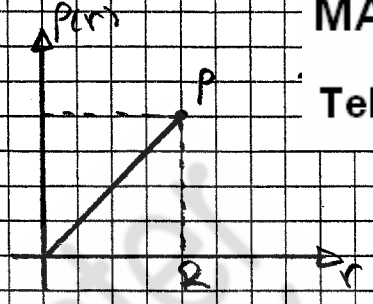


La distribuzione di carica dipende da r; il campo esterno rimane uguale alla sfera uniformemente carica, ma cambia il campo interno.

MASTER COPY
Tel. 388/9837745

Supponiamo di avere una densità lineare di ρ.

$$\rho(r) = \frac{\rho_0}{R} r$$



$$dq = \rho(r)\, dV \quad \text{(dV: volume della calotta sferica)}$$

$$dV = \frac{4}{3}\pi(r+dr)^3 - \frac{4}{3}\pi r^3 = \frac{4}{3}\pi\left(3r^2 dr + 3r(dr)^2 + (dr)^3\right)$$

$$dq = \rho(r)\, 4\pi r^2 dr$$

$$Q = \int dq = \int_0^R \rho(r)\, 4\pi r^2 dr = \int_0^R 4\pi \frac{\rho_0}{R} r^3 dr = \frac{4\pi\rho_0}{R}\int_0^R r^3 dr = \pi\rho_0 R^3 = \frac{4\pi\rho_0}{R}\cdot\frac{1}{4}R^4$$

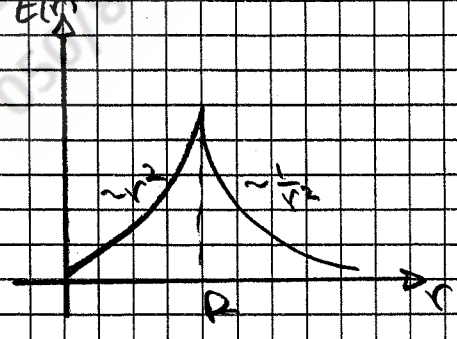
Per trovare il campo in un punto interno uso Gauss

$$\Phi = \frac{Q_{int}}{\varepsilon}$$

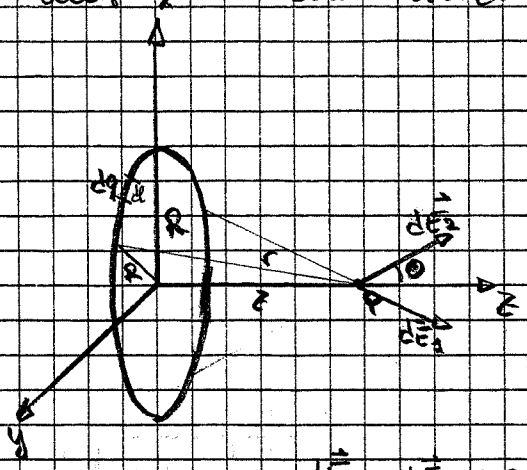
$$E_r\, 4\pi r^2 = \frac{\pi\rho_0}{R}\, r^4$$

$$\boxed{E_r = \frac{\rho_0}{4\varepsilon_0 R} r^2}$$

MASTER COPY
Tel. 388/9837745



Anello uniformemente carico



$dq = \lambda dl$ λ: densità lineare

$\int dq = Q$

$d\vec{E} = \frac{K dq \hat{r}}{r^3} = K \frac{dq}{r^2} \hat{r}$

Il campo elettrico sarà solo su z

MASTER COPY
Tel. 388/9837745

$d\vec{E}_z = d\vec{E} \cos\theta$

$d\vec{E}_z = K \frac{dq}{r^2} \cos\theta = K\lambda \frac{dl}{r^2} \cos\theta$

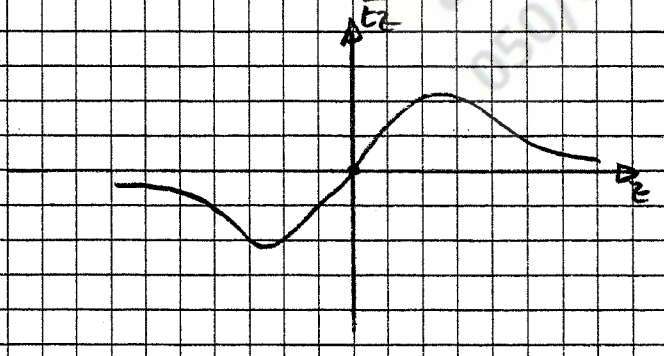
$E_z = K\lambda \int \frac{dl}{r^2} \cos\theta = K\lambda \frac{\cos\theta}{r^2} \int dl$ $\int dl = 2\pi R$

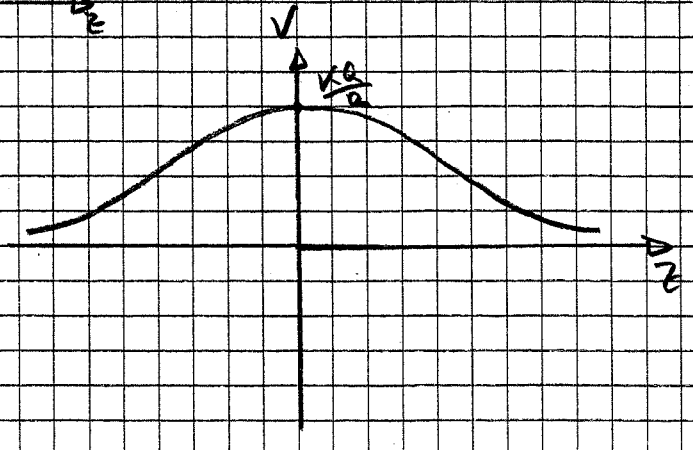
$E_z = K\lambda \frac{\cos\theta}{r^2} 2\pi R$ $r = \sqrt{z^2+R^2}$ $z = r\cos\theta \Rightarrow \cos\theta = \frac{z}{r}$

$E_z = K\lambda \frac{z}{r^3} 2\pi R = K\lambda \frac{z}{(z^2+R^2)^{\frac{3}{2}}} 2\pi R$ $E_z = KQ \frac{z}{(z^2+R^2)^{\frac{3}{2}}}$

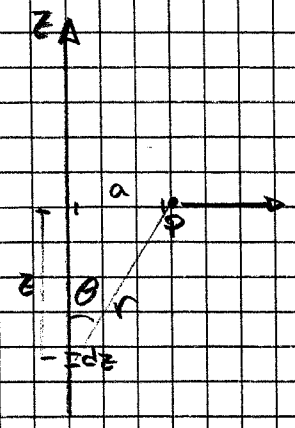
$dV = K\frac{dq}{r} = K \frac{dq}{\sqrt{z^2+R^2}}$

$V(z) = \int dV = \frac{K}{\sqrt{R^2+z^2}} \int dq = \frac{KQ}{\sqrt{z^2+R^2}}$ $V(z) = \frac{KQ}{\sqrt{z^2+R^2}}$





Filo



$$dq = \lambda dz$$

$$d\vec{E} = \frac{K dq}{r^2}\hat{r}$$

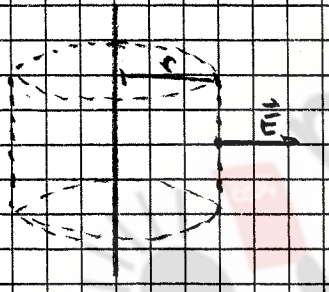
$$dE_x = \frac{K dq a}{r^3} = \frac{K \lambda dz\, a}{r^3}$$

$$E_x = \int_{-\infty}^{\infty} \frac{K\lambda a\, dz}{r^3} = K\lambda a \int_{-\infty}^{\infty} \frac{dz}{r(z)^3} = \int_{-\infty}^{\infty} \frac{dz}{(a^2+z^2)^{\frac{3}{2}}}$$

$$\vec{E} = \frac{2K\lambda}{a}\hat{r}$$

Potevo trovare lo stesso risultato con il teorema di Gauss

Sol. curva e superficie laterale perché sulle basi il flusso è 0 essendo il campo parallelo alle basi

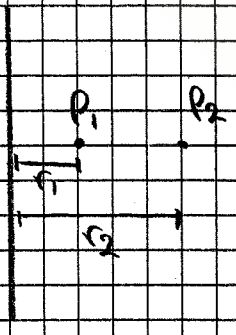


$$\Phi(\vec{E})_{\lambda_L} = \int E\, dA = E \int_{\lambda_L} dA = E\, 2\pi r h$$

$$Q_{INT} = \lambda h$$

$$\Phi(\vec{E}) = \frac{Q_{INT}}{\varepsilon} \qquad E\, 2\pi r K = \frac{\lambda K}{\varepsilon}$$

$$\boxed{E(r) = \frac{\lambda}{2\pi\varepsilon r}}$$



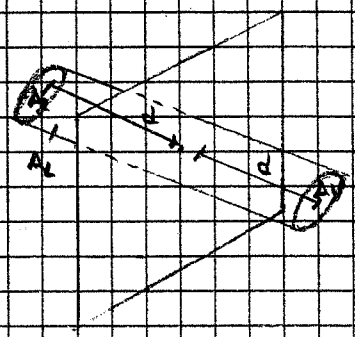
$$V(r_2) - V(r_1) = -\int_{r_1}^{r_2} \vec{E}\, d\vec{s} = -\int_{r_1}^{r_2} \frac{\lambda}{2\pi\varepsilon r}\hat{r}\, d\vec{s}$$

($d\vec{s} = dr$)

$$V(r_2) - V(r_1) = -\frac{\lambda}{2\pi\varepsilon}\int_{r_1}^{r_2} \frac{dr}{r} = -\frac{\lambda}{2\pi\varepsilon} \ln\left|\frac{r_2}{r_1}\right.$$

$$V(r_2) - V(r_1) = -\frac{\lambda}{2\pi\varepsilon} \ln\frac{r_2}{r_1}$$

Lastra infinita



$$\Phi(\vec{E}) = \Phi_{A_L} + \Phi_{A_1} + \Phi_{A_2}$$

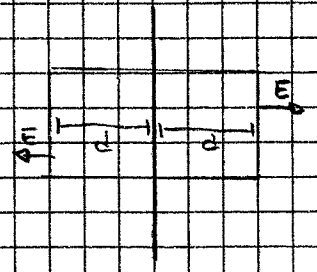
$$\Phi_{A_L} = 0$$

$$\Phi(\vec{E})_{A_1} = \int \vec{E}\,\hat{n}\,dA = E\cos\theta A_1 =$$

$$= EA_1$$

$$\Phi_{A_2} = EA_2$$

MASTER COPY
Tel. 388/9837745

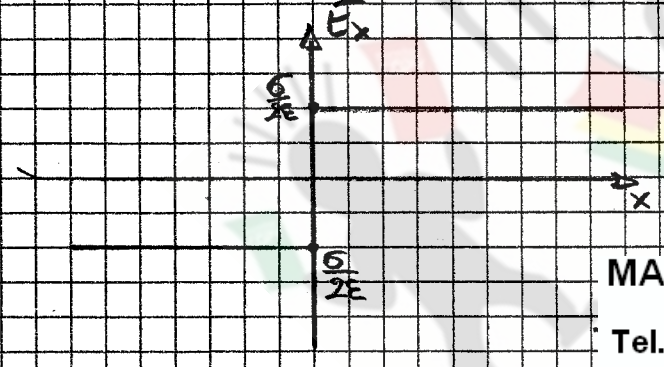


$$Q_{INT} = \sigma A_1$$

$$\Phi(\vec{E}) = \frac{Q_{INT}}{\varepsilon_0} \qquad \pm 2EA_1 = \pm \frac{|\sigma| A_1}{\varepsilon_0}$$

$$E = \frac{|\sigma|}{2\varepsilon} \qquad \vec{E} = \frac{\sigma}{2\varepsilon}\hat{x} \quad x > 0$$

$$\vec{E} = -\frac{\sigma}{2\varepsilon}\hat{x} \quad x < 0$$



MASTER COPY
Tel. 388/9837745

$$V_B - V_A = \int_A^B \vec{E}\,d\vec{s} = \vec{E}\int_A^B d\vec{s} = \vec{E}\cdot\vec{S}$$

Metto la 0 sul piano visto che non posso metterla all'infinito

$$V(x) = -Ex$$



Il potenziale decresce nel verso del campo

