Geometria e Algebra 1 – Esercizi del 6/12/07

(1) Risolvi in $\mathbb{C}$:

(a) $z^2 - (2+i)z + 3 + 11\cdot i = 0$

(b) $2z^3 + z^2 - 2z - 1 - i\cdot(z^2 + 6z - 7) = 0$

(c) $3z^4 - z^3 + 9z^2 - 4z - 12 - i(z^3 + z^2 + 16z + 4) = 0$

(d) $z^4\bar{z} - 2z^3 + |z|^2 = z$

(e) $z^5\bar{z} + z(z^3 + \bar{z}) = -1$

(2) Verifica la formula di Grassmann per le seguenti coppie di sottospazi complessi:

(a) $\{z \in \mathbb{C}^3 : z_1 + iz_2 = (1+i)z_3\}$

$\mathrm{Span}\left(\begin{pmatrix} 2 \\ -i \\ 1 \end{pmatrix}, \begin{pmatrix} 1-i \\ 1 \\ i \end{pmatrix}\right)$

(b) $\left\{ z \in \mathbb{C}^4 : \begin{array}{l} z_1 - iz_2 + 2z_3 - z_4 = 0 \\ (1+i)z_1 - 2z_2 + iz_3 - 2iz_4 = 0 \end{array} \right\}$

$\mathrm{Span}\left(\begin{pmatrix} ? \\ -i \\ 1+i \\ 0 \end{pmatrix}, \begin{pmatrix} -1 \\ 1-3i \\ 1-3i \\ 5 \end{pmatrix}\right)$

(3) Calcola

$\det\begin{pmatrix} i & 2-i \\ 7+i & 2+3i \end{pmatrix}$ $\qquad$ $\det\begin{pmatrix} -1 & i & 2+i \\ 2i & -3 & 1-i \\ -i & 1+2i & 2+i \end{pmatrix}$

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(4) Sia $f: \mathbb{R}^3 \to \mathbb{R}^2$ tale che

$$[f]_{\mathcal{B}}^{\mathcal{C}} = \begin{pmatrix} -1 & 2 & -3 \\ 3 & -7 & 1 \end{pmatrix}$$

dove $\mathcal{B} = \left(\begin{pmatrix} 3 \\ 2 \\ -3 \end{pmatrix}, \begin{pmatrix} 0 \\ 0 \\ 2 \end{pmatrix}, \begin{pmatrix} 8 \\ 5 \\ 8 \end{pmatrix} \right)$ $\mathcal{C} = \left(\begin{pmatrix} 4 \\ 7 \end{pmatrix}, \begin{pmatrix} 3 \\ 5 \end{pmatrix} \right)$;

determinare esplicitamente le coordinate di $f(x)$ per $x \in \mathbb{R}^3$.

(5) Siano $V = \left\{ x \in \mathbb{R}^4 : \begin{array}{l} 2x_1 - x_2 + x_3 = 0 \\ 3x_2 - x_3 + 2x_4 = 0 \end{array} \right\}$,

$\mathcal{B} = \left(\begin{pmatrix} 1 \\ 1 \\ -1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \\ 1 \\ -1 \end{pmatrix} \right)$. Al variare di $a \in \mathbb{R}$ si consideri

$$f_a : V \to \mathbb{R}^2 \qquad f_a(x) = \begin{pmatrix} x_1 + a x_2 \\ a x_3 + x_4 \end{pmatrix}$$

$$\mathcal{C}_a = \left(\begin{pmatrix} 2 \\ a \end{pmatrix} \begin{pmatrix} a \\ 2 \end{pmatrix} \right)$$

Si stabilisca per quali a $\mathcal{C}_a$ è base di $\mathbb{R}^2$ e per tali valori si scriva $[f_a]_{\mathcal{B}}^{\mathcal{C}_a}$. Si determini inoltre $\dim(\ker f_a)$ per ogni a.

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(4) Risolvi

$$\begin{cases} 2iz_1 + (2+3i)z_2 = 4 \\ (1-i)z_1 + (3+i)z_2 = 1-i \end{cases}$$

$$\begin{cases} (2-i)z_1 + 2iz_2 - 4z_3 = 5 \\ (1+3i)z_1 + z_2 + (1-i)z_3 = 1+i \end{cases}$$

(5) Trova equazioni cartesiane per

$$\left\{ \begin{pmatrix} z-i \\ 7+2i \end{pmatrix} + z \begin{pmatrix} -1+5i \\ z+3i \end{pmatrix} : z \in \mathbb{C} \right\}$$

$$\left\{ \begin{pmatrix} 1 \\ -i \\ 2 \end{pmatrix} + z \begin{pmatrix} 1+i \\ 2i \\ 3-4i \end{pmatrix} : z \in \mathbb{C} \right\}$$

$$\left\{ \begin{pmatrix} 1-i \\ 1+i \\ 2i \end{pmatrix} + z \begin{pmatrix} i \\ 2 \\ 1+i \end{pmatrix} + w \begin{pmatrix} -1 \\ 2-i \\ 3+2i \end{pmatrix} : z, w \in \mathbb{C} \right\}$$

(6) Verifica la formula delle dimensioni per

$$f : \mathbb{C}^3 \to \mathbb{C}^2$$

$$f(z) = \begin{pmatrix} iz_1 - 2z_2 + 4z_3 \\ (2-i)z_2 - iz_3 \end{pmatrix}$$

$$f : \mathbb{C}_{\leq 3}[z] \to \mathbb{C}_{\leq 2}[z]$$

$$f(p(z)) = p''(z) + p(i) \cdot z^2 + ip'(-1)$$

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Geometria e Algebra 1 – Esercizi dell'8/11/07

(1) Siano $V = \{x \in \mathbb{R}^3 : x_1 - 2x_2 + 5x_3 = 0\}$

$W = \{x \in \mathbb{R}^4 : 2x_1 - x_3 + x_4 = 0\}$

$\mathcal{B} = \left(\begin{pmatrix} -3 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 2 \\ 1 \end{pmatrix}\right)$

$\mathcal{C} = \left(\begin{pmatrix} 1 \\ 0 \\ 4 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ -1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 1 \\ 3 \\ -1 \end{pmatrix}\right)$

$\mathcal{D} = \left(\begin{pmatrix} 1 \\ 0 \\ -1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 2 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 2 \end{pmatrix}\right)$

$f : V \to W \qquad f(x) = \begin{pmatrix} x_1 + x_3 \\ 2x_1 - x_2 + x_3 \\ 3x_1 - x_2 \\ x_2 - 7x_3 \end{pmatrix}$

$g : W \to \mathbb{R}^3 \qquad g(x) = \begin{pmatrix} -x_1 + x_4 \\ x_2 - x_3 \\ 2x_2 - x_4 \end{pmatrix}$

Calcolare $[f]^{\mathcal{C}}_{\mathcal{B}}$, $[g]^{\mathcal{D}}_{\mathcal{C}}$, $[g \circ f]^{\mathcal{D}}_{\mathcal{B}}$

e verificare che $[g \circ f] = [g] \cdot [f]$.

(2) Sia $f : \mathbb{R}^2 \to \mathbb{R}^3$ lineare tale che

$f\begin{pmatrix} -2 \\ 7 \end{pmatrix} = \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix}$, $f\begin{pmatrix} -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 2 \\ 1 \\ -3 \end{pmatrix}$.

Determinare $[f]^{\mathcal{C}}_{\mathcal{B}}$ dove $\mathcal{B} = \left(\begin{pmatrix} 1 \\ 3 \end{pmatrix}\begin{pmatrix} 5 \\ -1 \end{pmatrix}\right)$

$\mathcal{C} = \left(\begin{pmatrix} 2 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} 2 \\ 3 \\ 0 \end{pmatrix}, \begin{pmatrix} 1 \\ 2 \\ 0 \end{pmatrix}\right)$.

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(3) Determinare, se esiste, l'inversa delle seguenti matrici:

$$\begin{pmatrix} 2 & -5 \\ 7 & -18 \end{pmatrix} \quad \begin{pmatrix} 1 & 2 & -4 \\ -1 & 3 & -11 \\ 4 & -1 & 11 \end{pmatrix} \quad \begin{pmatrix} 7 & 5 & 0 \\ 3 & 2 & 0 \\ -1 & 3 & 1 \end{pmatrix}$$

(4) Sia $f: \mathbb{R}^2 \to \mathbb{R}^2$ data da $f(x) = \begin{pmatrix} 2x_1 - 3x_2 \\ -x_1 + 4x_2 \end{pmatrix}$.

Verificare le formule di cambio di base con per $[f]$

$$\mathcal{B} = \left(\begin{pmatrix} 1 \\ 4 \end{pmatrix}, \begin{pmatrix} -1 \\ -3 \end{pmatrix} \right)$$

$$\mathcal{C} = \left(\begin{pmatrix} 2 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right)$$

$$\mathcal{B}' = \left(\begin{pmatrix} 6 \\ 17 \end{pmatrix}, \begin{pmatrix} 1 \\ 3 \end{pmatrix} \right)$$

$$\mathcal{C}' = \left(\begin{pmatrix} 5 \\ 3 \end{pmatrix}, \begin{pmatrix} 7 \\ 4 \end{pmatrix} \right)$$

(5) Sia $V = \{ x \in \mathbb{R}^3 : 2x_1 - x_2 + 5x_3 = 0 \}$

$$r = \begin{pmatrix} 2 \\ -1 \\ -1 \end{pmatrix}$$

$$\mathcal{B} = \left(\begin{pmatrix} 1 \\ 7 \\ 1 \end{pmatrix}, \begin{pmatrix} 3 \\ 1 \\ -1 \end{pmatrix} \right)$$

$$\mathcal{B}' = \left(\begin{pmatrix} 4 \\ 3 \\ -1 \end{pmatrix}, \begin{pmatrix} -5 \\ 0 \\ 2 \end{pmatrix} \right)$$

Verificare le formule di cambio di base per $[r]$.

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