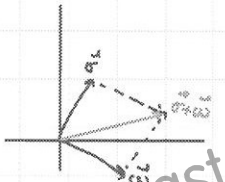
$\vec{v} - \vec{\omega} = \vec{v} + (-1)\vec{\omega}$

$\lambda > 1 \rightarrow$ dilatazione
$\lambda < 1 \rightarrow$ contrazione
$\lambda < 0 \rightarrow$ ribalton. rispetto all'origine.

$\vec{v} = (1, 2)$

$\vec{\omega} = (2, -1)$

$\vec{v} + \vec{\omega} = (3, 1)$



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PRODOTTO SCALARE:

$\vec{x} = (x_1, \dots x_n) \qquad \vec{y} = (y_1 \dots y_n)$

$\vec{x} \cdot \vec{y} = \langle \vec{x}, \vec{y} \rangle = x_1 y_1 + x_2 y_2 + \dots + x_n y_n$

PROPRIETÀ PRODOTTO SCALARE:

① $\langle \vec{x}, \vec{y} + \vec{z} \rangle = \langle \vec{x}, \vec{y} \rangle + \langle \vec{x}, \vec{z} \rangle$

mm. $x_1(y_1 + z_1) + x_2(y_2 + z_2) + \dots + x_n(y_n + z_n)$

$= x_1 y_1 + x_1 z_1 + x_2 y_2 + x_2 z_2 + \dots + x_n y_n + x_n z_n$

$= \langle \vec{x}, \vec{y} \rangle + \langle \vec{x}, \vec{z} \rangle$

② $\langle \lambda \vec{x}, \vec{y} \rangle = \lambda x_1 y_1 + \lambda x_2 y_2 + \dots + \lambda x_n y_n$

$= \lambda \langle \vec{x}, \vec{y} \rangle$

$= \lambda \langle \vec{y}, \vec{x} \rangle$

NORMA O LUNGHEZZA DI UN VETTORE:

$\vec{x} = (x_1 \dots x_n)$

NORMA: $\|\vec{x}\| = |\vec{x}| = \sqrt{x_1^2 + \dots + x_n^2} = \sqrt{\langle \vec{x}, \vec{x} \rangle}$

lunghezza di un vettore.

DISTANZA FRA 2 VETTORI:

$\vec{x} = (x_1 \dots x_n) \qquad \vec{y} = (y_1 \dots y_n)$

$d(\vec{x}, \vec{y}) = \|\vec{x} - \vec{y}\| = \sqrt{(x_1 - y_1)^2 + \dots + (x_n - y_n)^2}$

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NORMA DI SOMMA E DIFFERENZA:

$\|\vec{x} + \vec{y}\|^2 = \langle \vec{x} + \vec{y}, \vec{x} + \vec{y} \rangle$

$= \langle \vec{x} + \vec{y}, \vec{x} \rangle + \langle \vec{x} + \vec{y}, \vec{y} \rangle$

$= \langle \vec{x}, \vec{x} \rangle + \langle \vec{y}, \vec{x} \rangle + \langle \vec{x}, \vec{y} \rangle + \langle \vec{y}, \vec{y} \rangle$

$= \|\vec{x}\|^2 + 2\langle \vec{x}, \vec{y} \rangle + \|\vec{y}\|^2$

$\|-1 \vec{x}\| = |-1| \, \|\vec{x}\|$

$\|\vec{x} - \vec{y}\|^2 = \langle x - y, x - y \rangle$

$= \langle x - y, x \rangle + \langle x - y, -y \rangle$

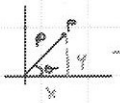
$= \langle x, x \rangle + -\langle y, x \rangle + -\langle x, y \rangle + \langle y, y \rangle$

$= \|\vec{x}\|^2 + \|\vec{y}\|^2 - 2\langle \vec{x}, \vec{y} \rangle$

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COORDINATE POLARI NEL PIANO.

ρ, θ



$\begin{cases} x = \rho\cos\theta \\ y = \rho\sin\theta \end{cases}$ $\quad \rho = \sqrt{x^2+y^2}$ = NORMA DEL VETTORE $\quad \tan\theta = \frac{y}{x}$

$x = \rho\cos\alpha$ $\quad \frac{y}{x} = \frac{\rho\sin\theta}{\rho\cos\theta} = \tan\theta$ ✓
$y = \rho\sin\alpha$

SIGNIFICATO GEOMETRICO PRODOTTO SCALARE:

$\vec{x} = (x_1, x_2)$ $\longrightarrow$ $x_1 = \rho\cos\theta$ $\quad x_2 = \rho\sin\theta$
$\vec{y} = (y_1, y_2)$ $\quad y_1 = \varrho\cos\varphi$ $\quad y_2 = \varrho\sin\varphi$

$\longrightarrow \langle \vec{x}, \vec{y} \rangle = \|\vec{x}\| \|\vec{y}\| \cos(\text{angolo compreso})$

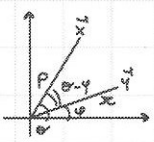
$\langle \vec{x}, \vec{y} \rangle = x_1y_1 + x_2y_2$

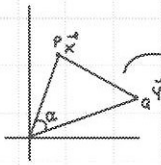
$= \rho\cos\theta\,\varrho\cos\varphi + \rho\sin\theta\,\varrho\sin\varphi$

$= \rho\varrho(\cos\theta\cos\varphi + \sin\theta\sin\varphi)$

$= \rho\varrho\cos(\theta-\varphi)$

$= \|\vec{x}\| \cdot \|\vec{y}\| \cos(\theta-\varphi)$





$\|\vec{x}-\vec{y}\|$ $\quad OP = \|\vec{x}\|$
$\quad OQ = \|\vec{y}\|$
$\quad PQ = \|\vec{x}-\vec{y}\|$

* TEOREMA COSENO:

$PQ^2 = OP^2 + OQ^2 - 2OP \cdot OQ \cos\alpha$

$\|\vec{x}-\vec{y}\|^2 = \|\vec{x}\|^2 + \|\vec{y}\|^2 - 2\|\vec{x}\|\|\vec{y}\|\cos\alpha$

$= \|\vec{x}\|^2 + \|\vec{y}\|^2 - 2\langle \vec{x}, \vec{y} \rangle \quad \Rightarrow \langle \vec{x}, \vec{y} \rangle = \|\vec{x}\| \cdot \|\vec{y}\| \cos\alpha.$

CONSEGUENTE:

1. $\langle \vec{x}, \vec{y} \rangle = 0 \iff \cos(\theta-\varphi) = 0$
$\iff \vec{x} \perp \vec{y}$

2. $|\langle \vec{x}, \vec{y} \rangle| \leq \|\vec{x}\| \cdot \|\vec{y}\|$ $\quad$ DISUGUAGLIANZA DI CAUCHY-SCHWARZ.

Dim. $|\langle \vec{x}, \vec{y} \rangle| = |\|\vec{x}\| \cdot \|\vec{y}\| \cdot \cos(\dots)|$

$= \|\vec{x}\| \cdot \|\vec{y}\| \, |\cos(\dots)|$
$\qquad\qquad\qquad\qquad\qquad\qquad \downarrow \leq 1$

$\leq \|\vec{x}\| \cdot \|\vec{y}\|$

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DISUGUAGLIANZA DI CAUCHY-SCHWARZ:

$$|\langle \vec{x}, \vec{y} \rangle| \leq \|\vec{x}\| \cdot \|\vec{y}\|$$

Dim 1) $\forall T \in \mathbb{R}$ considero $\|\vec{x} + T\vec{y}\|^2 \geq 0$ sviluppo: $\|\vec{x}\|^2 + \|T\vec{y}\|^2 + 2\langle \vec{x}, T\vec{y} \rangle$

$\|\vec{x}\|^2 + T^2\|\vec{y}\|^2 + 2T\langle \vec{x}, \vec{y} \rangle$

$= \|\vec{y}\|^2 T^2 + 2\langle \vec{x}, \vec{y} \rangle T + \|\vec{x}\|^2$

$(AT^2 + 2BT + C \geq 0 \ \forall T \in \mathbb{R}) \Rightarrow \Delta \leq 0$

$\Delta \leq 0 \quad B' - AC \leq 0 \quad B'^2 \leq AC$

$\langle \vec{x}, \vec{y} \rangle^2 \leq \|\vec{x}\|^2 \cdot \|\vec{y}\|^2$

$|\langle \vec{x}, \vec{y} \rangle| \leq \|\vec{x}\| \cdot \|\vec{y}\|$

Dim 2) $|\langle \vec{x}, \vec{y} \rangle| \leq \|\vec{x}\| \cdot \|\vec{y}\|$

$\langle \vec{x}, \vec{y} \rangle = \|\vec{x}\| \cdot \|\vec{y}\| \cdot \cos(\sigma - \varphi)$

$|\langle \vec{x}, \vec{y} \rangle| = \|\vec{x}\| \cdot \|\vec{y}\| \cdot |\cos(\ldots)| \leq \|\vec{x}\| \cdot \|\vec{y}\|$ ($|\cos(\ldots)| \leq 1$)

$\langle \vec{x}, \vec{y} \rangle = x_1 y_1 + x_2 y_2$

$\vec{x} = (x_1, x_2) \rightarrow$ c. polari $\quad x_1 = \rho \cos\sigma \quad x_2 = \rho \sin\sigma$

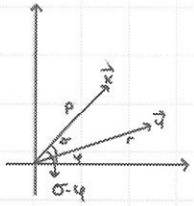
$\vec{y} = (y_1, y_2) \quad y_1 = r\cos\varphi \quad y_2 = r \sin\varphi$

$\langle \vec{x}, \vec{y} \rangle = \rho r \cos\sigma \cos\varphi + \rho r \sin\sigma \sin\varphi$

$= \rho r (\cos\sigma\cos\varphi + \sin\sigma\sin\varphi)$

$= \rho r \cos(\sigma - \varphi)$

$= \|\vec{x}\| \cdot \|\vec{y}\| \cdot \cos(\sigma - \varphi)$



DISUG. TRIANGOLARE

$$\|\vec{x} + \vec{y}\| \leq \|\vec{x}\| + \|\vec{y}\|$$

$\|\vec{x} + \vec{y}\|^2 = \langle (\vec{x}+\vec{y}), (\vec{x}+\vec{y}) \rangle$

$\langle \vec{x}, \vec{x}+\vec{y} \rangle + \langle \vec{y}, \vec{x}+\vec{y} \rangle$

$\langle \vec{x}, \vec{x} \rangle + \langle \vec{x}, \vec{y} \rangle + \langle \vec{y}, \vec{x} \rangle + \langle \vec{y}, \vec{y} \rangle$

$\|\vec{x}\|^2 + 2\langle \vec{x}, \vec{y} \rangle + \|\vec{y}\|^2$

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RETTA → RAPPRESENTAZIONE CARTESIANA:
- esplicita: $y = mx + q$ → non si rappresentano $x = \alpha$. RETTE VERTICALI!! $m = \tan\alpha = \frac{y}{x}$
- implicita: $ax + by + c = 0$ → rappresentazione non è unica, → non occupa una tripla (a, b, c)

→ RAPPRESENTAZIONE PARAMETRICA $(a, b) + T(c, d) = (a + Tc, b + Td)$

Retta che passa x 2 punti $P \neq Q$: $P + T(Q - P)$ → direzione retta
$Q - T(P - Q)$ → stessa retta percorsa al contrario.

$(a, b) + T(c, d) \rightarrow m = \frac{d}{c}$ se rappresento in cartesiana dico come esprimo in verticale di d

$c = 0$: retta verticale.

CARTESIANA → PARAMETRICA

1. calcolo 2 punti a mano = $P + T(Q - P)$

PARAMETRICA → CARTESIANA

$(2, 3) + T(1, -2)$

1. sostituisco $T = 0$ trovo 2 punti e $y - y_1 = m(x - x_1)$
   $T = 1$

2. scrivo la parametrica come $(2 + T, 3 - 2T)$
   $x \quad y$

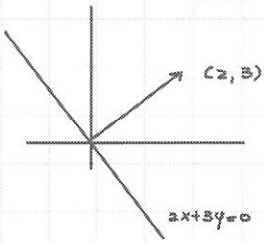
$\begin{cases} 2 + T = x & T = x - 2 \\ y = 3 - 2T & y = 3 - 2x = 3 - 2(x - 2) = 3 - 2x + 4 \end{cases} \Rightarrow y = 7 - 2x$

COSENO ANGOLO COMPRESO TRA 2 VETTORI? $|\langle \vec{x}, \vec{y} \rangle| = \|\vec{x}\| \cdot \|\vec{y}\| \cos(\text{angolo compreso})$

RETTE PASSANTI: $y = mx$
X L'ORIGINE $ax + by = 0 \rightsquigarrow \langle (a,b), (x,y) \rangle = 0$ (Tutti i vettori $(x,y) \perp (a,b)$)

$ax+by \Longrightarrow$ (a,b) = Tutti i vettori $\perp$ alla retta.

nella
immagine



$y = -\frac{2}{3}x$

$(0,0) + T(3,-2)$

$(0,0) + T(-3,2)$

$ax + by = 0 \Longrightarrow (0,0) + T(-b,a) \quad (0,0) + T(b,-a) \Rightarrow$ direzione = vettore $\perp$ al vettore (a,b)

$Ax + By + C = 0$

↓

vettore (A,B) = vettore $\perp$ alla direzione della retta. $\longrightarrow$ possibile direzione = $(-B, A)$