ANALISI 1

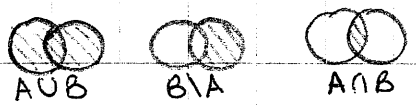
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INSIEME O CLASSE

TEORIA $B := \{x \in \mathbb{N} : x \text{ è pari}\} = \{0, 2, 4, 6 \dots\}$

$\emptyset$ insieme vuoto $A \in B$ $x \in A$ $x \subset A$ (contenuto)

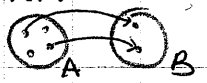
$A \cap B$ è un insieme che ha elementi che sono contenuti sia in A che in B $A \cap B := \{x \in A \text{ e } x \in B \ \forall x\}$



FUNZIONI

INIETTIVA: associa elementi diversi a elementi diversi

SURGETTIVA: $\forall b \in B$ è un immagine di $a \in A$



BIGETTIVA: Sia iniettiva che surgettiva $\forall a \in A \ \exists$ uno e uno solo $b \in B$

BIUNIVOCA: Se è inversa $g(f(a)) = a \ \forall a \in A$ ∄ per tutte le funz $y^{-1} \ \exists \Leftrightarrow \exists$ y bigettiva!

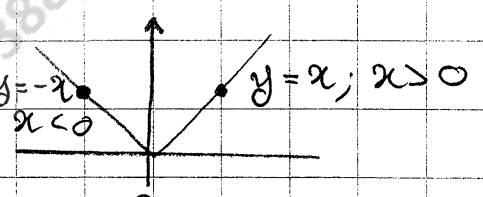
EQUIPOTENTI Se A e B hanno lo stesso numero di elementi

INSIEMI

$\mathbb{N}$ (naturali) $\subset \mathbb{Z}$ (interi) $\subset \mathbb{Q}$ (razionali) $\subset \mathbb{R}$ (reali)

VALORE ASSOLUTO

$x \in \mathbb{R}$ $\quad |x| := \begin{cases} x & \text{Se } x \geq 0 \\ -x & \text{Se } x < 0 \end{cases}$



$\text{Graf}|\cdot| = \{(x, y);\ x \in \mathbb{R};\ y \in \mathbb{R};\ y |x|\}$

PROPRIETÀ

① $|x+y| \leq |x| + |y|$

② $|x-y| \geq |x| - |y|$

③ $|xy| = |x|\,|y|$

MAGGIORANTE $A \subset \mathbb{R}$ $M \in \mathbb{R}$ $\forall a \in A$ $a \leq M$ M è maggiorante

MINORANTE $A \subset \mathbb{R}$ $m \in \mathbb{R}$ $\forall a \in A$ $a \geq m$ m è minorante

Un insieme è limitato superiormente se ∃ almeno un M.

Un insieme è limitato inferiormente se ∃ almeno un m.

MASSIMO:

$A \subset \mathbb{R}$ $\forall a \in A$ $\exists M \in \mathbb{R}$ t.c. $a \leq M$ e $M \in A$ $M = \max A$

MINIMO:

$A \subset \mathbb{R}$ $\forall a \in A$ $\exists m \in \mathbb{R}$ t.c. $a \geq m$ e $m \in A$ $m = \min A$

$A \subset \mathbb{R}$ $A \neq \emptyset$ $\sup A = +\infty$ non è limitato sup

$A \subset \mathbb{R}$ $A \neq \emptyset$ $\inf A = -\infty$ non è limitato inf

TEOREMA SULLA CARATTERIZZAZIONE DI ESTREMO INF. E SUP.

1. $A \subset \mathbb{R}$ $A \neq \phi$ è lim. sup. $M = \sup A$ o M è mag
   - $\forall \varepsilon > 0$ $\exists a \in A$ $M - \varepsilon \leq a \leq M$
2. $A \subset \mathbb{R}$ $A \neq \emptyset$ è lim. inf. $m = \inf A$ o m è min
   - $\forall \varepsilon > 0$ $\exists a \in A$ $m \leq a \leq m + \varepsilon$

FUNZIONI

$f: \mathbb{R} \to \mathbb{R}$ $D(f) = \text{Dominio}(f)$

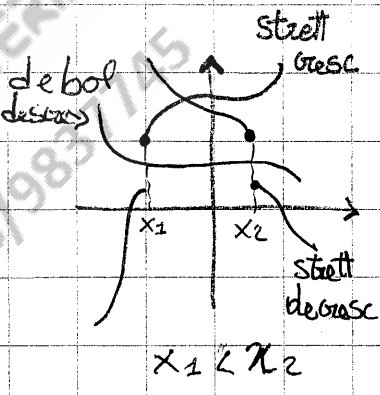
PROPRIETA' $f: \mathbb{R} \to \mathbb{R}$

Strett. cresc. Se $x_1 < x_2 \Rightarrow f(x_1) < f(x_2)$

Strett. decresc. Se $x_1 > x_2 \Rightarrow f(x_1) > f(x_2)$

Non cresc. Se $x_1 \leq x_2 \Rightarrow f(x_1) \leq f(x_2)$

Non decresc. Se $x_1 \geq x_2 \Rightarrow f(x_1) \geq f(x_2)$



- DISPARI $f(-x) = -f(x)$ Simmetrico rispetto $(0,0)$
- PARI $f(-x) = f(x)$ Simmetrico rispetto asse y
- FUNZIONE SEGNO $f(x) = \text{Segno}(x) = \text{sign}(x)$ $\begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$
- PERIODICA $f: (x+T) = f(x)$ $T > 0$ T = periodo

FUNZIONI ELEMENTARI

FUNZIONE POTENZIALE $f(x) = x^n$ $n \in \mathbb{N}$ $n = 0$ $x \neq 0$

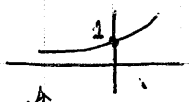
$f(x) = x^{-n}$ $n \in \mathbb{N}$ $n = 0$ $x \leq 0$ $D(f) = \mathbb{R} \setminus \{0\}$

$\Rightarrow$ $f(x) = x^{\alpha}$ $\alpha \in \mathbb{R}$ $D(f) = \{x : \forall x > 0\}$

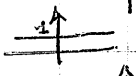
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FUNZIONE ESPONENZIALE $f(x)=a^x$ $a>0$ né pari né dispari

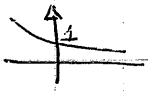
CASO 1 $a>1$ $x=0$ $D:\mathbb{R}$



CASO 2 $a=1$ $D:\mathbb{R}$



CASO 3 $0<a<1$ $D:\mathbb{R}$



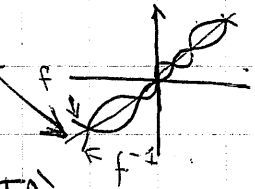
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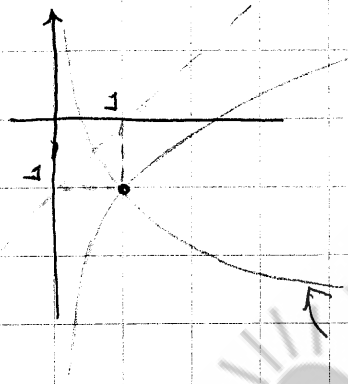
FUNZIONE LOGARITMICA

a^x $a>0$ $a\neq 1$ $g(x)=a^x$ $a^x=y \Rightarrow x=\log_a y$

$g^{-1}(y)=x$; $g(x)=y$

Il grafico della funzione inversa è uguale a quella di partenza ribaltata, tracciando una bisettrice





$\log_a x$ $a\in(1,+\infty)$ Sempre crescente

$\log_a x$ $a\in(0,1)$ Sempre decrescente

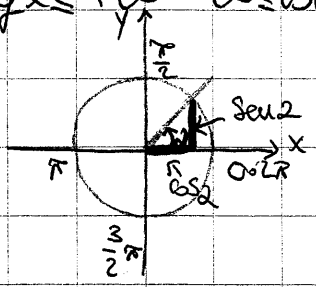
PROPRIETÀ

- $\log_a x = -\log_{1/a} x$
- $\log_a \frac{x}{y} = \log_a x - \log_a y$
- $\log_a xy = \log_a x + \log_a y$
- $\log_a a = 1$ $\log_a 1 = 0$
- $\lg x = \log_{10} x$

FUNZIONI TRIGONOMETRICHE

$-1\leq \text{Sen}\alpha\leq 1$ $-1\leq \cos\alpha\leq 1$ $-\infty\leq tg x\leq +\infty$ $-\infty\leq \text{cotg}x\leq+\infty$

$tg\alpha = \text{Sen}\alpha/\cos\alpha$ $\text{cotg}\alpha = \cos\alpha/\text{Sen}\alpha$



DOMINIO

$D(f=\text{Sen}\alpha)=\mathbb{R}$ $D(f=\cos\alpha)=\mathbb{R}$

$D(f=tg\alpha)=\mathbb{R}\setminus\{\frac{\pi}{2}+k\pi\ ,\ k\in\mathbb{Z}\}$

$D(f=\text{cotg}\alpha)=\mathbb{R}\setminus\{k\pi;\ k\in\mathbb{Z}\}$

$\text{Sen}(-x)=-\text{Sen}x$ → funzione dispari

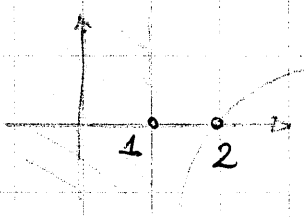
$\cos(-x)=\cos x$ → funzione pari

ESERCIZI SUL GRAFICO DI FUNZIONE

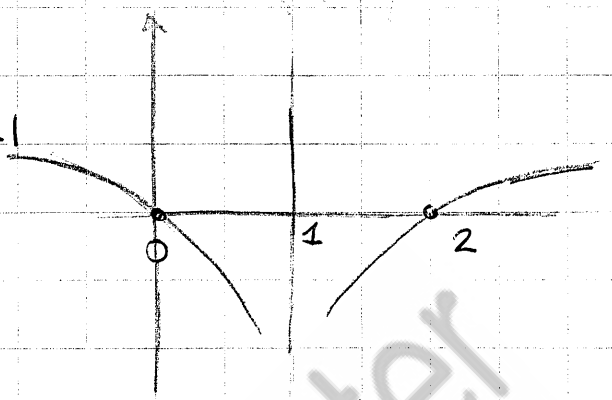
- $y = |2 \log|x-1||$

$g(x) = \log|x-1|$

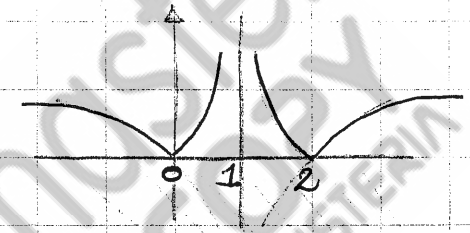
$x > 1$



$h(x) = 2\log|x-1|$

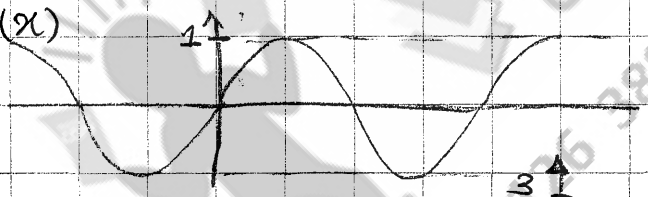


$f(x) = |2\log|x-1||$

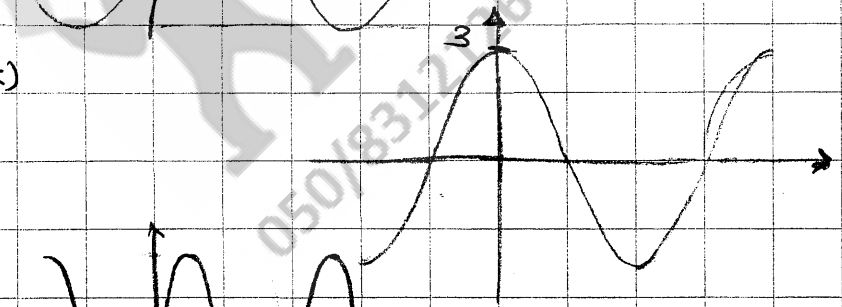


- $y = |2\,\text{Sen}(3x) - 1|$

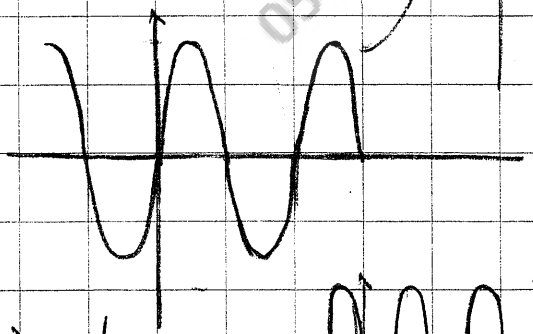
$g(x) = \text{Sen}(x)$



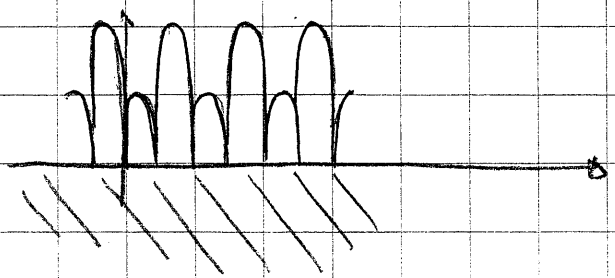
$h(x) = \text{Sen}(3x)$



$l(x) = 2\,\text{Sen}(3x)$



$f(x) = |2\,\text{Sen}(3x) - 1|$



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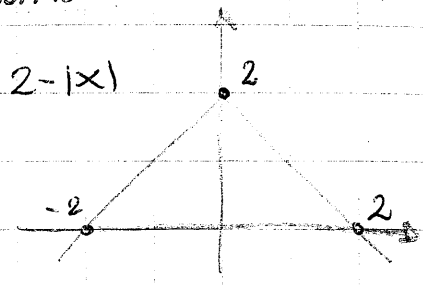
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- $y = |3\sqrt{2-|x|} - 2|$

$g(x) = |x|$

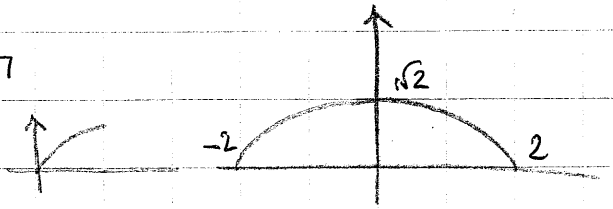


$h(x) = 2-|x|$

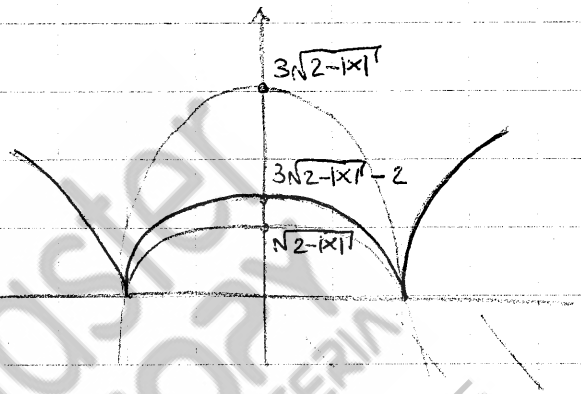


$q(x) = \sqrt{2-|x|}$

$m(x) = \sqrt{2-x}$



$f(x) = |3\sqrt{2-|x|} - 2$



ESERCIZIO SUL DOMINIO

$f(x) = \sqrt{5^{4x} + 2\sqrt{6}(5^{2x}) - 19}$

Dominio posto $5^{2x} = y$

$y^2 + 2\sqrt{6}\,y - 19 \geq 0 \qquad \Delta = 24 + 76 = 100 \qquad y_{1,2} = \begin{cases} -\sqrt{6}+5 \\ -\sqrt{6}-5 \end{cases}$

$y \leq -\sqrt{6} - 5 \quad \vee \quad y \geq -\sqrt{6} + 5$

non accett. mai verificata $\nexists x$

$5^{2x} \geq -\sqrt{6} + 5$

$2x \geq \log_5(-\sqrt{6}+5) \Rightarrow x \geq \frac{1}{2}\log_5(-\sqrt{6}+5)$

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