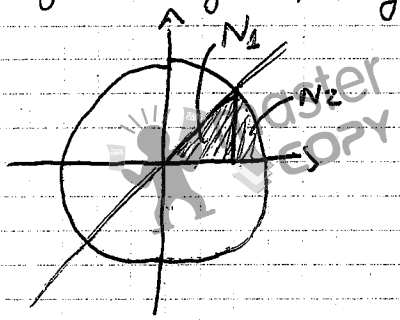
- $x^2+y^2 \leq 1$
- $\{(x,y) \mid -1 \leq x \leq 1, \ -\sqrt{1-x^2} \leq y \leq \sqrt{1-x^2}\}$
- $\{(x,y) \mid -1 \leq y \leq 1, \ -\sqrt{1-y^2} \leq x \leq \sqrt{1-y^2}\}$

- $\Omega = \{(x,y) \mid x^2+y^2 \leq 1, \ y \leq x, \ x \geq 0, \ y \geq 0\}$



- $N_1 = \{(x,y) \mid 0 \leq x \leq \frac{1}{\sqrt{2}} \quad 0 \leq y \leq x\}$

$N_2 = \{(x,y) \mid \frac{1}{\sqrt{2}} \leq x \leq 1, \ 0 \leq y \leq \sqrt{1-x^2}\}$

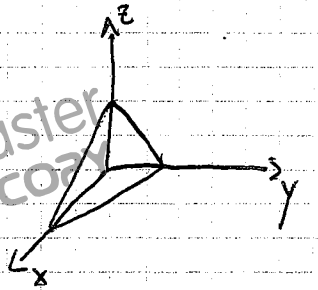
- $N = \{(x,y) \mid 0 \leq y \leq \frac{1}{\sqrt{2}}, \ y \leq x \leq \sqrt{1-y^2}\}$

- $T = \{$triangolo di vertici $(0,0), (0,1), (1,1)\}$
  - $T = \{(x,y) \mid 0 \leq x \leq 1, \ x \leq y \leq 1\}$
  - $T = \{(x,y) \mid 0 \leq y \leq 1 \quad 0 \leq x \leq y\}$

- $T = \{$piramide di vertici $(0,0,0), (1,0,0), (0,1,0), (0,0,1)\}$

① per fili

$$\iint_B dx\,dy \left(\int_0^{1-x-y} f(x,y,z)\,dz \right)$$

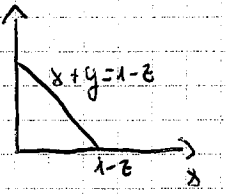


$B = \{$triangolo in (x,y) di vertici $(0,0), (0,1), (1,0)\}$

② per sezioni

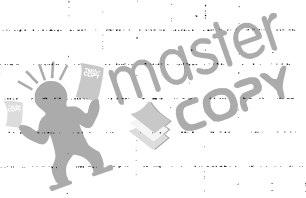
$$T = \{(x,y,z) \mid 0 \leq z \leq 1, \ (x,y) \in T_z\}$$

$T_z \rightarrow$

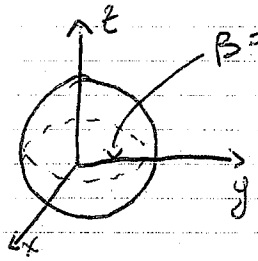


$$\int_0^1 dz \left(\int_{T_z} dx\,dy \right)$$


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- $\mathrm{Vol}(B_r^3(0))$ $\qquad B_r^3(0) = \{x^2+y^2+z^2 \le r^2\}$



$B = \{(x,y) \mid x^2+y^2 \le r^2\}$

- $B_r^3(0) = \{(x,y) \mid -\sqrt{r^2-x^2-y^2} \le z \le \sqrt{r^2-x^2-y^2}$

① Fili

$$\iint_B \left(\int_{-\sqrt{r^2-x^2-y^2}}^{\sqrt{r^2-x^2-y^2}} 1\,dz \right) dx\,dy$$

- $-r \le z \le r$ $\qquad \Omega_z = \{(x,y) \mid x^2+y^2 \le r^2-z^2\}$

② Sezioni

$$\int_{-r}^{r} dz \left(\iint_{x^2+y^2 \le r^2-z^2} 1\,dx\,dy \right)$$

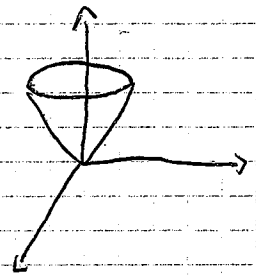
- $\Omega = \{(x,y,z) \mid x^2+y^2 \le z \le 4\}$

① Fili $\qquad \iint_B dx\,dy \int_{x^2+y^2}^{4} dz$ $\qquad B = \{(x,y) \mid x^2+y^2 \le 4\}$

② Sezioni $\qquad \int_0^4 dz \left(\iint_{-2z} dx\,dy \right)$ $\qquad \Omega_z = \{(x,y) \mid x^2+y^2 \le z\}$

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- $\Omega = \{(x,y,z) \mid x^2+y^2+z^2 \le 1 \ , \ z \ge \sqrt{x^2+y^2}\}$



per fili $\quad x^2+y^2 \le 1-x^2-y^2$

$x^2+y^2 \le \frac{1}{2}$

$\sqrt{x^2+y^2} \le z \le \sqrt{1-x^2-y^2}$

$$\iint_{x^2+y^2 \le \frac{1}{2}} dx\,dy \left(\int_{\sqrt{x^2+y^2}}^{\sqrt{1-x^2-y^2}} dz \right)$$

per sezioni

$0 < z < 1 \qquad 1-z^2 = z^2 \qquad z = \frac{1}{\sqrt{2}}$

$\Omega = \Omega_1 + \Omega_2$

$\Omega_1 = \{ x^2+y^2 \le z^2 \quad \text{con } z \in [0, \frac{1}{\sqrt{2}}]$

$\Omega_2 = \{ x^2+y^2 \le 1-z^2 \quad \text{con } z \in [\frac{1}{\sqrt{2}}, 1]$

- Semisfera superiore

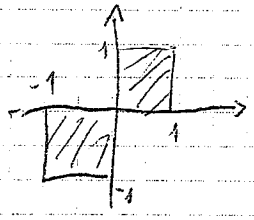
$\{(x,y,z) \mid x^2+y^2 \le R^2 \ , \ z = \sqrt{R^2-x^2-y^2}$

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• $\Omega = x^2+y^2+z^2 < 1 \quad , \quad x>0$

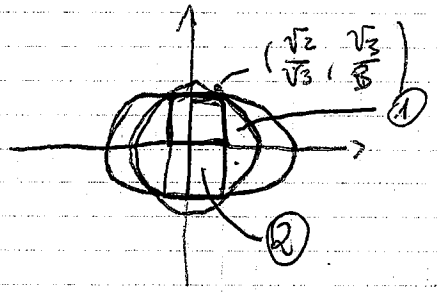
$\theta \in [-\frac{\pi}{2}, \frac{\pi}{2}] \qquad \varphi \in [0, \frac{\pi}{2}] \qquad \rho \in [0,1]$

• $\Omega = \{(x,y) \mid \min\{x,y\} > -1, \ \max\{x,y\} < 1, \ xy > 0\}$



• $A \cap B \qquad A = \{(x,y) \mid x^2+y^2 \le 1\}$

$B = \{(x,y) \mid \frac{1}{2}x^2 + 2y^2 \le 1\}$



①

$$4\left(\int_{\frac{\sqrt{2}}{\sqrt{3}}}^{1} dx \int_{0}^{\sqrt{1-x^2}} dy\right) + 2\left(\int_{-\frac{\sqrt{2}}{\sqrt{3}}}^{\frac{\sqrt{2}}{\sqrt{3}}} dx \int_{-\sqrt{\frac{1-\frac{1}{2}x^2}{2}}}^{\sqrt{\frac{1-\frac{1}{2}x^2}{2}}} dy\right)$$